WORKSHOP CALCULATION & SCIENCE

(NSQF)

(As per Revised Syllabus July 2022)

Domestic Painter

Directorate General of Training

DIRECTORATE GENERAL OF TRAINING
MINISTRY OF SKILL DEVELOPMENT & ENTREPRENEURSHIP
GOVERNMENTOF INDIA

NATIONAL INSTRUCTIONAL MEDIA INSTITUTE, CHENNAI

Post Box No. 3142, CTI Campus, Guindy, Chennai - 600 032

Workshop Calculation & Science

Domestic Painter - 1 Year NSQF

As per Revised Syllabus July 2022

Developed & Published by

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FOREWORD

The Government of India has set an ambitious target of imparting skills one out of every four Indians, to help them secure jobs as part of the National Skills Development Policy. Industrial Training Institutes (ITIs) play a vital role in this process especially in terms of providing skilled manpower. Keeping this in mind, and for providing the current industry relevant skill training to Trainees, ITI syllabus has been recently updated with the help of comprising various stakeholder's viz. Industries, Entrepreneurs, Academicians and representatives from ITIs.

The National Instructional Media Institute (NIMI), Chennai, has now come up with instructional material to suit the revised curriculum for **Workshop Calculation & Science - Domestic Painter** NSQF (Revised 2022) under CTS will help the trainees to get an international equivalency standard where their skill proficiency and competency will be duly recognized across the globe and this will also increase the scope of recognition of prior learning. NSQF trainees will also get the opportunities to promote life long learning and skill development. I have no doubt that with NSQF the trainers and trainees of ITIs, and all stakeholders will derive maximum benefits from these IMPs and that NIMI's effort will go a long way in improving the quality of Vocational training in the country.

The Executive Director & Staff of NIMI and members of Media Development Committee deserve appreciation for their contribution in bringing out this publication.

Jai Hind

Director General (Training),
Ministry of Skill Development & Entrepreneurship,
Government of India.

New Delhi - 110 001

PREFACE

The National Instructional Media Institute(NIMI) was set up at Chennai, by the Directorate General of Training, Ministry of skill Development and Entrepreneurship, Government of India, with the technical assistance from the Govt of the Federal Republic of Germany with the prime objective of developing and disseminating instructional Material for various trades as per prescribed syllabus and Craftsman Training Programme(CTS) under NSQF levels.

The Instructional materials are developed and produced in the form of Instructional Media Packages (IMPs), consisting of Trade Theory, Trade Practical, Test and Assignment Book, Instructor Guide. The above material will enable to achieve overall improvement in the standard of training in ITIs.

A national multi-skill programme called SKILL INDIA, was launched by the Government of India, through a Gazette Notification from the Ministry of Finance (Dept of Economic Affairs), Govt of India, dated 27th December 2013, with a view to create opportunities, space and scope for the development of talents of Indian Youth, and to develop those sectors under Skill Development.

The emphasis is to skill the Youth in such a manner to enable them to get employment and also improve Entrepreneurship by providing training, support and guidance for all occupation that were of traditional types. The training programme would be in the lines of International level, so that youths of our Country can get employed within the Country or Overseas employment. The **National Skill Qualification Framework (NSQF)**, anchored at the National Skill Development Agency(NSDA), is a Nationally Integrated Education and competency-based framework, to organize all qualifications according to a series of **levels of Knowledge, Skill and Aptitude.** Under NSQF the learner can acquire the Certification for Competency needed at any level through formal, non-formal or informal learning.

The **Workshop Calculation & Science** - Domestic Painter NSQF (Revised 2022) under CTS is one of the book developed by the core group members as per the NSQF syllabus.

The **Workshop Calculation & Science** - Domestic Painter NSQF (Revised 2022) under CTS as per NSQF is the outcome of the collective efforts of experts from Field Institutes of DGT, Champion ITI's for each of the Sectors, and also Media Development Committee **(MDC)** members and Staff of **NIMI.** NIMI wishes that the above material will fulfill to satisfy the long needs of the trainees and instructors and shall help the trainees for their Employability in Vocational Training.

NIMI would like to take this opportunity to convey sincere thanks to all the Members and Media Development Committee (MDC) members.

Chennai - 600 032 **EXECUTIVE DIRECTOR**

ACKNOWLEDGEMENT

The National Instructional Media Institute (NIMI) sincerely acknowledge with thanks the co-operation and contribution of the following Media Developers to bring this IMP for the course **Workshop Calculation & Science - Domestic Painter** as per NSQF Revised 2022.

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Shri. G. Michael Johny	-	Manager, NIMI, Chennai - 32.

NIMI records its appreciation of the **Data Entry, CAD, DTP Operators** for their excellent and devoted services in the process of development of this IMP.

NIMI also acknowledges with thanks, the efforts rendered by all other staff who have contributed for the development of this book.

INTRODUCTION

The material has been divided into independent learning units, each consisting of a summary of the topic and an assignment part. The summary explains in a clear and easily understandable fashion the essence of the mathematical and scientific principles. This must not be treated as a replacment for the instructor's explanatory information to be imparted to the trainees in the classroom, which certainly will be more elaborate. The book should enable the trainees in grasping the essentials from the elaboration made by the instructor and will help them to solve independently the assignments of the respective chapters. It will also help them to solve the various problems, they may come across on the shop floor while doing their practical exercises.

The assignments are presented through 'Graphics' to ensure communications amongst the trainees. It also assists the trainees to determine the right approach to solve the problems. The required relevent data to solve the problems are provided adjacent to the graphics either by means of symbols or by means of words. The description of the symbols indicated in the problems has its reference in the relevant summaries.

At the end of the exercise wherever necessary assignments, problems are included for further practice.

Time allotment:

Duration of 1 Year : 18 Hrs

Time allotment for each title of exercises has been given below. **Workshop Calculation & Science - Domestic Painter** NSQF Revised Syllabus 2022.

S.No	Title	Exercise No.	Time in Hrs
1	Unit, Fractions	1.1.01 - 1.1.07	4
2	Square root, Ratio and Proportions, Percentage	1.2.08 - 1.2.14	6
3	Mensuration	1.3.15 - 1.3.19	6
4	Trigonometry	1.4.20 & 1.4.21	2
		Total	**18 Hrs**

LEARNING / ASSESSABLE OUTCOME

On completion of this book you shall be able to

- **Demonstrate basic mathematical concept and principles to perform practical operations.**
- **Understand and explain basic science in the field of study.**

CONTENTS

SYLLABUS

1 Year

Workshop Calculation & Science - Domestic Painter
Revised syllabus July 2022 under CTS

S.No.	Title	Time in Hrs
I	**Unit, Fractions** 1 Classification of Unit System 2 Fundamental and Derived Units F.P.S, C.G.S, M.K.S and SI Units 3 Measurement Units and Conversion 4 Factors, HCF, LCM and Problems 5 Fractions – Addition, Subtraction, Multiplication & Division 6 Decimal Fractions – Addition, Subtraction, Multiplication & Division 7 Solving Problems by using calculator	4
II	**Square root, Ratio and Proportions, Percentage** 1 Square and Square root 2 Simple problems using calculator 3 Applications of Pythagoras theorem and related problems 4 Ratio and Proportion 5 Ratio and Proportion - Direct and Indirect proportions 6 Percentage 7 Percentage - Changing percentage to decimal and fraction	6
III	**Mensuration** 1 Area and perimeter of square, rectangle and parallelogram 2 Area and Perimeter of triangles 3 Area and Perimeter of circle, semi-circle, circular ring, sector of circle, hexagon and ellipse 4 Surface area and Volume of solids - cube, cuboid, cylinder, sphere and hollow cylinder 5 Finding the lateral surface area, total surface area and capacity in litres of hexagonal, conical and cylindrical shaped vessels	6
IV	**Trigonometry** 1 Measurement of angles 2 Trigonometrical ratios	2
	Total	**18**

Unit, Fractions - Classification of unit system

Necessity

All physical quantities are to be measured in terms of standard quantities.

Unit

A unit is defined as a standard or fixed quantity of one kind used to measure other quantities of the same kind.

Classification

Fundamental units and derived units are the two classifications.

Fundamental units

Units of basic quantities of length, mass and time.

Derived units

Units which are derived from basic units and bear a constant relationship with the fundamental units.E.g. area, volume, pressure, force etc.

Systems of units

– F.P.S system is the British system in which the basic units of length, mass and time are foot, pound and second respectively.

– C.G.S system is the metric system in which the basic units of length, mass and time are centimeter, gram and seconds respectively.

– M.K.S system is another metric system in which the basic units of length, mass and time are metre, kilogram and second respectively.

– S.I. units are referred to as Systems International units which is again of metric and the basic units, their names and symbols are as follows.

Fundamental units and derived units are the two classifications of units.

Length, mass and time are the fundamental units in all the systems (i.e) F.P.S, C.G.S, M.K.S and S.I. systems.

Example

Length: What is the length of copper wire in the roll , if the roll of copper wire weighs 8kg, the dia of wire is 0.9cm and the density is 8.9 gm/cm^3?

Solution

mass of copper wire in the roll = 8kg (or)8000grams
Dia of copper wire in the roll = 0.9cm
Density of copper wire = 8.9 gm/cm^3

Area of cross section of copper wire

$$= \frac{\pi d^2}{4} = \frac{\pi \times (0.9^2)}{4} = 0.636cm^2$$

Volume of copper wire

$$= \frac{\text{Mass of copper wire}}{\text{Density of copper wire}} = \frac{8000\text{grams}}{8.9\ \text{gm/cm}^3} = 898.88\text{cm}^3$$

Length of copper wire

$$= \frac{\text{Volume of copper wire}}{\text{Area of cross section of copper wire}} = \frac{898.88\text{cm}^3}{0.636\text{cm}^2}$$

$$= 1413.33 \text{ cm}$$

Length of copper wire =1413cm.

Time: The S.I. unit of time, the second, is another base units of S.I., it is defined as the time interval occupied by a number of cycles of radiation from the calcium atom. The second is the same quantity in the S.I. in the British and in the U.S. systems of units.

Fundamental units of F.P.S, C.G.S, M.K.S and S.I

S.No.	Basic quantity	British units		Metric units				International units	
		F.P.S	Symbol	C.G.S	Symbol	M.K.S	Symbol	S.I Units	Symbol
1	Length	Foot	ft	Centimetre	cm	Metre	m	Metre	m
2	Mass	Pound	lb	Gram	g	Kilogram	kg	Kilogram	Kg
3	Time	Second	s	Second	s	Second	s	Second	s
4	Current	Ampere	A	Ampere	A	Ampere	A	Ampere	A
5	Temperature	Fahrenheit	°F	Centigrade	°C	Centigrade	°C	Kelvin	K
6	Light intensity	Candela	Cd	Candela	Cd	Candela	Cd	Candela	Cd

Unit, Fractions - Fundamental and Derived units F.P.S, C.G.S, M.K.S and SI units

Derived units of F.P.S, C.G.S, M.K.S and SI system

S.No	Physical quantity	British units		Metric units				International units	
		FPS	Symbol	CGS	Symbol	MKS	Symbol	SI Units	Symbol
1	Area	Square foot	ft^2	Square centimetre	cm^2	Square metre	m^2	Square metre	m^2
2	Volume	Cubic foot	ft^3	Cubic centimetre	cm^3	Cubic metre	m^3	Cubic metre	m^3
3	Density	Pound per cubic foot	lb/ft^3	Gram per cubic centimetre	g/cm^3	Kilogram per cubic metre	kg/m^3	Kilogram per cubic metre	Kg/m^3
4	Speed	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
5	Velocity (linear)	Foot per second	ft/s	Centimetre per second	cm/sec	Metre per second	m/sec	Metre per second	m/sec
6	Acceleration	Foot per square second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre per square second	m/sec^2
7	Retardation	Foot per square Second	ft/s^2	Centimetre per square second	cm/sec^2	Metre per square second	m/sec^2	Metre square second	m/sec^2
8	Angular velocity	Degree per second	Deg/sec	Radian per second	rad/sec	Radian per second	rad/sec	Radian per second	rad/sec
9	Mass	Pound (slug)	lb	Gram	g	Kilogram	kg	Kilogram	kg
10	Weight	Pound	lb	Gram	g	Kilogram weight	kg	Newton	N
11	Force	Pounds	lbf	dyne	dyn	Kilogram force	kgf	Newton	$N(kgm/sec^2)$
12	Power	Foot pound per second	ft.lb/sec	Gram.centimetre/sec	g.cm/sec	kilogram metre per second	kg.m/sec	-	-
		Horse power	hp	Erg per second		watt	W	watt	W(J/sec)
13	Pressure,Stress	Pound per square inch	lb/in^2	Gram per square centimetre	g/cm^2	Kilogram per square metre	kg/m^2	Newton per square metre	N/m^2
14	Energy, Work	Foot.pound	ft.lb	Gram centimetre	g.cm	Kilogram metre	kg.m	joule	J(Nm)
15	Heat	British thermal unit	BTU	calorie	Cal	joule	J	joule	J(Nm)
16	Torque	Pound force foot	lbf.ft	Newton millimetre	N mm	Kilogram metre	kg.m	Newton metre	Nm
17	Temperature	Degree Fahrenheit	°F	Degree Centigrade	°C	Kelvin	K	Kelvin	K

Unit, Fractions - Measurement units and conversion

Units and abbreviations

Quantity	Units	Abbreviation of unit
Calorific value	kilojoules per kilogram	kJ/kg
Specific fuel consumption	kilogram per hour per newton	kg/hr/N
Length	millimetre, metre, kilometre	mm, m, km
Mass	kilogram, gram	kg, g
Time	seconds, minutes, hours	s, min, h
Speed	centimetre per second, metre per second kilometre per hour, miles per hour	cm/s, m/s km/h, mph
Acceleration	metre-per-square second	m/s^2
Force	newtons, kilonewtons	N,kN
Moment	newton-metres	Nm
Work	joules	J
Power	horsepower, watts, kilowatts	Hp, W, kW
Pressure	newton per square metre kilonewton per square metre	N/m^2 kN/m^2
Angle	radian	rad
Angular speed	radians per second radians-per-square second revolutions per minute revolutions per second	rad/s rad/s^2 Rpm rev/s

Decimal multiples and parts of unit

Decimal power	Value	Prefixes	Symbol	Stands for
10^{12}	1000000000000	tera	T	billion times
10^{9}	1000000000	giga	G	thousand millintimes
10^{6}	1000000	mega	M	million times
10^{3}	1000	kilo	K	thousand times
10^{2}	100	hecto	h	hundred times
10^{1}	10	deca	da	ten times
10^{-1}	0.1	deci	d	tenth
10^{-2}	0.01	centi	c	hundredth
10^{-3}	0.001	milli	m	thousandth
10^{-6}	0.000001	micro	μ	millionth
10^{-9}	0.000000001	nano	n	thousand millionth
10^{-12}	0.000000000001	pico	p	billionth

SI units and the British units:

Quantity	SI unit → British unit	British unit → SI unit
Length	1 m = 3.281 ft 1 km = 0.621 mile	1 ft = 0.3048 m 1 mile = 1.609 km
Speed	1 m/s = 3.281 ft/s 1 km/h = 0.621 mph	1 ft/s = 0.305 m/s 1 mph = 1.61 km/h
Acceleration	1 m/s^2 = 3.281 ft/s^2	1 ft/s^2 = 0.305 m/s^2
Mass	1 kg = 2.205 lb	1 lb = 0.454 kg
Force	1 N = 0.225 lbf 1 MN	1 lbf = 4.448 N 1 million newtons
Torque	1 Nm = 0.738 lbf ft	1 lbf ft = 1.355 Nm
Pressure	1 N/m^2 = 0.000145 lbf/in^2 1 Pa = 1 N/m^2 1 bar = 14.5038 lbf/in^2	1 lbf/in^2 = 6.896 kN/m^2 1 lbf/in^2 = 6.895 kN/m^2
Energy, work	1 J = 0.738 ft lbf 1 J = 0.239 calorie 1 kJ = 0.948 BTU (1 therm = 100 000 BTU) 1 kJ = 0.526 CHU	1 ft lbf = 1.355 J 1 calorie = 4.186 J 1 BTU = 1.055 kJ 1 CHU = 1.9 kJ
Power	1 kW = 1.34 hp	1 hp = 0.7457 kW
Fuel consumption	1km/L = 2.82 mile/gallon	1 mpg = 0.354 km/L
Specific fuel consumption	1 kg/kWh = 1.65 lb/bhp h 1 litre/kWh=1.575 pt/bhp h	1 lb/bhp h = 0.606 kg/kWh 1 pt/bhp h = 0.631 litre/kWh
Calorific value	1 kJ/kg = 0.43 BTU/lb 1 kJ/kg = 0.239 CHU/lb	1 BTU/lb = 2.326 kJ/kg 1 CHU/lb = 4.188 kJ/kg

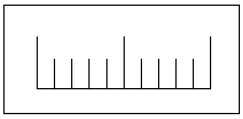
Prefixes for decimal multiples and submultiples

Use		
1 Megapascal	= 1 MPa	= 1000000 Pa
1 Kilowatt	= 1 kW	= 1000 W
1 Hectolitre	= 1 hL= 100 L	
Decanewton	= 1 daN	= 10 N
Decimetre	= 1 dm	= 0.1 m
1 Centimetre	= 1 cm	= 0.01 m
1 Millimetre	= 1 mm	= 0.001 m
1 Micrometre	= 1 um	= 0.000001 m

Conversion factors

1 inch	=	25.4 mm
1 mm	=	0.03937 inch
1 metre	=	39.37 inch
1 micron	=	0.00003937"
1 kilometre	=	0.621 miles
1 pound	=	453.6 g
1 kg	=	2.205 lbs
1 metric ton	=	0.98 ton

Units of physical quantities

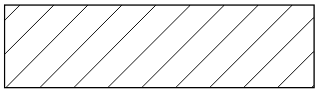


UNITS OF LENGTH

Units of length

Micron	1 μ	=	0.001 mm
Millimetre	1 mm	=	1000 μ
Centimetre	1 cm	=	10 mm
Decimetre	1 dm	=	10 cm
Metre	1 m	=	10 dm
Kilometre	1 km	=	1000 m
Inch	1"	=	25.4 mm
Foot	1'	=	0.305 m
Yard	1 Yd	=	0.914 m
Nautical mile	1 NM	=	1852 m
Geographical mile	1	=	1855.4 m

Units of area



UNITS OF AREA

Square millimetre	1 mm^2	
Square centimetre	1 cm^2	= 100 mm^2
Square decimetre	1 dm^2	= 100 cm^2
Square metre	1 m^2	= 100 dm^2
Are	1 a	= 100 m^2
Hectare	1 ha	= 100 a
Square kilometre	1 km^2	= 100 ha
Square inch	1 sq.in	= 6.45 cm^2
Square foot	1 sq.ft	= 0.093 m^2
Square yard	1 sq.yd	= 0.84 m^2
Square metre	1 m^2	= 10.76 ft^2
Acre	1	= 40.5 a

1 Acre = 100 cent

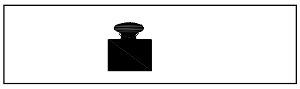
1 Cent = 436 Sq. ft.
1 Ground = 2400 Sq.ft.

1 Hectare = 2.47 acres

1 acre = 0.4047 Hectare

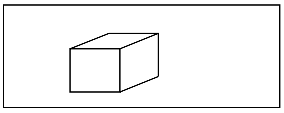
1 Hectare = 10000 sq. metre

Units of weight



UNITS OF WEIGHT

Milligram - force	1 mgf	
Gram-force	1 gf	1000 mgf
Kilogram-force	1 kgf	= 1000 gf
Tonne	1 t	= 1000 kgf
Ounce	1	= 28.35 gf
Pound	1 lbs	= 0.454 kgf
Long ton	1	= 1016 kgf
Short ton	1	= 907 kgf

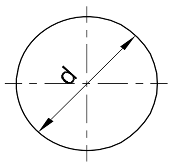


UNITS OF VOLUME AND CAPACITY

Units of volume and capacity

Cubic millimetre	1 mm^3	
Cubic centimetre	1 cm^3	= 1000 mm^3
Cubic decimetre	1 dm^3	= 1000 cm^3
Cubic metre	1 m^3	= 1000 dm^3
Litre	1 l	= 1 dm^3
Hectolitre	1 hl	= 100 l
Cubic inch	1 cu. in	= 16.387 cm^3
Cubic foot	1 cu. ft	= 28317 cm^3
Gallon (British)	1 gal	= 4.54 l
1cubic metre	1 m^3	= 1000 litres
1000 Cu.cm	1000 cm^3	= 1 litre
1 cubic foot	1 ft^3	= 6.25 Gallon
1 litre	1lt	= 0.22 Gallon

Circular unit



CIRCULAR OF UNIT

Radian

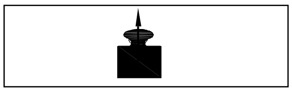
Relationship between Radian and Degree

1 Radian $= \frac{180^\circ}{\pi}$

$180^0 = \pi$ Radian;

1 Degree $= \frac{\pi}{180}$ Radian

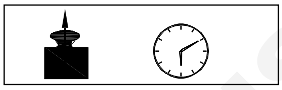
Work



WORK

Kilogram-force	1 kgfm	= 9.80665 J
Metre	1 kgfm	= 9.80665 Ws
Joule	1 J	= 1 Nm
Watt-second	1 Ws	= 0.102 kgfm
Kilowatt hour	1 kWh	= 3.6 x 10^6 J
		= 859.8456 $kcal_{IT}$
I.T.Kilocalorie	1 $kcal_{IT}$	= 426.kgfm

Power



POWER

Kilogram-force metre/second

1 kgfm/s	= 9.80665 W
Kilowatt	1 kW = 1000 W = 1000 J/s
	= 102 kgfm/s (approx.)
Metric horse power	1 HP = 75 kgfm/s
	= 0.736 kW
1 Calorie	= 4.187J
I.T.Kilocalorie/hour = 1 $kcal_{IT/h}$ = 1.163 W	

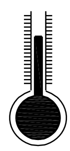
Pressure

Pascal	1 Pa	= 1 N/m^2	1 atm	= 101325 Pa
Bar	1 bar = 10N/cm^2	= 100000 Pa–Torr	1 torr	$= \frac{101325}{760} \approx 133.32$ pa
Atmosphere	1 atm	= 1 kgf/cm^2	1 kgf/cm^2 = 735.6 mm of mercury	

TEMPERATURE

Scale	Freezing point	Boiling point
Centigrade (˚C)	0°C	100°C
Fahrenheit (°F)	32°F	212°F
Kelvin (K)	273K	373K
Reaumur (°R)	0°R	80°R

TEMPERATURE



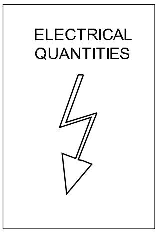
$$\frac{^\circ R}{80} = \frac{^\circ C}{100} = \frac{K-273}{100} = \frac{^\circ F-32}{180}$$

FORCE

Force	In C.G.S. System : Force (Dyne)	= Mass (gm)XAcceleration (cm/sec^2)
	In F.P.S. System : Force (Poundal)	= Mass (lb) X Acceleration (ft./sec^2)
	In M.K.S System : Force (Newton)	= Mass (Kg) x Acceleration (mtr./sec^2)
	1 Dyne	= 1 gm x1 cm/sec^2
	1 Poundal	= 1 lb x 1 ft/sec^2
	1 Newton	= 1 kg x 1 mtr/sec^2 = 10^5 dynes
	1gm weight	= 981 Dynes
	1 lb weight	= 32 Poundals
	1 kg weight	= 9.81 Newtons

ELECTRICAL QUANTITIES

V	Electric potential	V	Volt	V(W/A)
E	Electromotive force	V	Volt	V(W/A)
I	Electric current	A	Ampere	A
R	Electric resistance	Ω	Ohm	Ω (V/A)
e	Specific resistance	Ω m	Ohm metre	Vm/A
G	Conductance	Ω^{-1}	Siemens	S



Assignment - Answer the following question.

1 Convert 320 kilometres into miles ______

2 Convert 16 tons into kilograms ______

3 Convert 40 inches into centimetres ______

4 Convert 8 metres into feet ______

5 Convert 2.5 gallons into litres ______

6 Convert 5 litres into gallons ______

7 120°C = ______ °F.

8 Expand the abbreviations of the following

a N/m^2

b RPM

9 Convert the following S.I. units as required.

a Length

i 3.4 m = ______mm

ii 10.2 km = ______mile

b Mass

i 650 g = ______kg

ii 120 mg = ______g

c Force

i 1.2 N = ______kg

ii 25 kg = ______N

d Work, energy, amount of heat

i 120 KJ = ______J

ii 300 wh = ______kwh

e Power

i 0.2 kW = ______W

ii 350 W = ______kW

f Convert as required.

i 5 N = ______KN

Unit, Fractions - Factors, HCF, LCM and problems

Prime Numbers and whole Numbers

Factor

A factor is a small number which divides exactly into a bigger number.e.g.

To find the factors of 24, 72, 100 numbers

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The numbers 2,3,5 are called factors.

Definition of a prime factor

Prime factor is a number which divides a prime number into factors.e.g.

57 = 3 x 19

The numbers 3 and 19 are prime factors.

They are called as such, since 3 & 19 also belong to prime number category.

Definition of H.C.F

The Highest Common Factor

The H.C.F of a given group of numbers is the highest number which will exactly divide all the numbers of that group.e.g.

To find the H.C.F of the numbers 24, 72, 100

24 = 2 x 2 x 2 x 3

72 = 2 x 2 x 2 x 3 x 3

100 = 2 x 2 x 5 x 5

The factors common to all the three numbers are

2 x 2 = 4. So HCF = 4.

Definition of L.C.M

Lowest common multiple

The lowest common multiple of a group of numbers is the smallest number that will contain each number of the given group without a remainder.e.g.

- Factorise the following numbers

  7,17,20,66,128

  7,17 - These two belong to Prime numbers. Hence no factor except unity and itself.

2	20
2	10
	5

Factors of 20 = 2 x 2 x 5

2	66
3	33
	11

Factors of 66 = 2 x 3 x 11

2	128
2	64
2	32
2	16
2	8
2	4
	2

Factors of 128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

- Select prime numbers from 3 to 29

  3,5,7,11,13,17,19,23,29

- Find the HCF of the following group of numbers HCF of 78, 128, 196

2	78
3	39
	13

78 = 2 x 3 x 13

2	128
2	64
2	32
2	16
2	8
2	4
	2

128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

2	196
2	98
	49

196 = 2 x 2 x 49

HCF = 2

- Find LCM of 84,92,76

LCM =

2	84, 92, 76
2	42, 46, 38
3	21, 23, 19
	7, 23, 19

LCM = 2 x 2 x 3 x 7 x 23 x 19 = 36708

- To find out the LCM of 36, 108, 60

2	36, 108, 60
2	18, 54, 30
3	9, 27, 15
3	3, 9, 5
	1, 3, 5

LCM of the number

36, 108, 60 = 2 x 2 x 3 x 3 x 3 x 5 = 540

The necessity of finding LCM and HCF arises in subtraction and addition of fractions.

Unit, Fractions - Fractions - Addition, subtraction, multiplication & division

Description

A minimal quantity that is not a whole number. For e.g. .

$\frac{1}{5}$ a vulgur fraction consists of a numerator and denominator.

Numerator/Denominator

The number above the line in a vulgar fraction showing how many of the parts indicated by the denominator are taken is the numerator. The total number of parts into which the whole quantity is divided and written below the line in a vulgar fraction is the denominator. e.g.

$\frac{1}{4}, \frac{3}{4}, \frac{7}{12}$

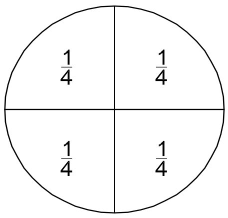
1,3,7 - numerators 4,12 - denominators

Fraction: Concept

Every number can be represented as a fraction.e.g.

$1\frac{1}{4} = \frac{5}{4}$, A full number can be represented as an apparent fraction.e.g. (Fig 1)

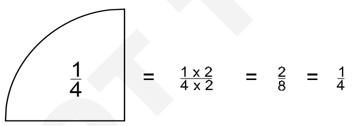
Fig 1



Fraction: Value

The value of a fraction remains the same if the numerator and denominator of the fraction are multiplied or divided by the same number.(Fig 2)

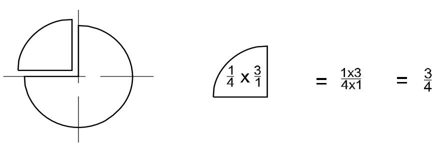
Fig 2



Multiplication

When fractions are to be multiplied, multiply all the numerators to get the numerator of the product and multiply all the denominators to form the denominator of the product. (Fig 3)

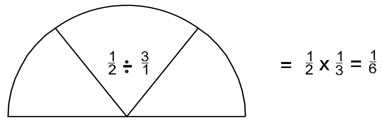
Fig 3



Division

When a fraction is divided by another fraction the dividend is multiplied by the reciprocal of the divisor. (Fig 4)

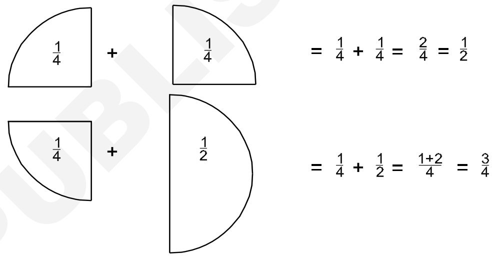
Fig 4



Addition and Subtraction

The denominators of the fractions should be the same when adding or subtracting the fractions. Unequal denominators must first be formed into a common denominator. It is the lowest common denominator and it is equal to the product of the most common prime numbers of the denominators of the fractions in question.(Fig 5)

Fig 5



Examples

- Multiply $\frac{3}{4}$ by $\frac{2}{3}$,

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

- Divide $\frac{3}{8}$ by $\frac{3}{4}$,

$$\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} x \frac{4}{3} = \frac{1}{2}$$

- Add $\frac{3}{4}$ and $\frac{2}{3}$,

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

- sub $\frac{7}{16}$ from $\frac{17}{32}$

$$\frac{17}{32} - \frac{7}{16} = \frac{17}{32} - \frac{14}{32} = \frac{(17-14)}{32} = \frac{3}{32}$$

Types of fractions

- Proper fractions are less than unity. Improper fractions have their numerators greater than the denominators.
- A mixed number has a full number and a fraction.

Addition of fraction

Add $\frac{1}{2}+\frac{1}{8}+\frac{5}{12}$

To add these fractions we have to find out L.C.M of denominators 2,8,12.

Find L.C.M of 2,8,12

Step 1 L.C.M

2	2,8,12
2	1,4,6
	1,2,3

Factors are 2,2,2,3

Hence L.C.M = 2 x 2 x 2 x 3 = 24

Step 2

$\frac{1}{2}+\frac{1}{8}+\frac{5}{12}=\frac{12}{24}+\frac{3}{24}+\frac{10}{24}$

$=\frac{12+3+10}{24}=\frac{25}{24}=1\frac{1}{24}.$

Subtraction of fraction

subtract $9\frac{15}{32}$ from $17\frac{9}{16}$ or $(17\frac{9}{16}-9\frac{15}{32})$

Step 1: Subtract whole number first 17 - 9 = 8

Step 2: L.C.M of 16,32 = 32

Since number 16 divides the number 32

Subtracting fractions = $\frac{3}{32}$

Adding with whole number from Step 1

we get $8+\frac{3}{32}=8\frac{3}{32}$

Common fractions

Problems with plus and minus sign

Example

solve $3\frac{3}{4}+6\frac{7}{8}-4\frac{5}{16}-\frac{9}{32}$

Rule to be followed

1 Add all whole numbers

2 add all + Numbers

3 Add all - Numbers

4 Find L.C.M of all denominators

Solution

Step 1: Add whole numbers = 3 + 6 - 4 = 5

Step 2: Add fractions = $\frac{3}{4}+\frac{7}{8}-\frac{5}{16}-\frac{9}{32}$

L.C.M of 4,8,16,32 is 32

$\frac{24+28-10-9}{32}$

$=\frac{52-19}{32}$

$=\frac{33}{32}=1\frac{1}{32}$

Step 3: Adding again with the whole number

we get $5+1\frac{3}{32}=6\frac{3}{32}$

Examples

Common fractions

- Multiply

a $\frac{3}{8}$ by $\frac{4}{7}=\frac{3}{8}$ x $\frac{4}{7}=\frac{3}{14}$ b $\frac{2}{3}$ x $\frac{3}{4}$ x $\frac{5}{8}=\frac{5}{16}$

- Division

a $\frac{5}{16}\div\frac{5}{32}=\frac{5}{16}$ x $\frac{32}{5}=2$

b $4\frac{2}{3}\div3\frac{1}{7}=\frac{14}{3}\div\frac{22}{7}=\frac{14}{3}$ x $\frac{7}{22}=\frac{49}{33}=1\frac{16}{33}$

- Addition

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

L..C.M = 2,4,8 = 8

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{4+2+1}{8}=\frac{7}{8}$

- Subtraction

$5\frac{1}{4}-3\frac{3}{4}=5-3+\frac{1}{4}-\frac{3}{4}$

$=2+\frac{1}{4}-\frac{3}{4}=2\frac{1}{4}-\frac{3}{4}$

$=\frac{9}{4}-\frac{3}{4}=\frac{9-3}{4}$

$=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$

Assignment

1 Convert the following into improper fractions.

a $1\frac{2}{7}$ = ________

b $4\frac{3}{5}$ = ________

c $3\frac{3}{5}$ = ________

2 Convert the following into mixed numbers.

a $\frac{12}{11}$ = ________

b $\frac{36}{14}$ = ________

c $\frac{18}{10}$ = ________

3 Place the missing numbers.

a $\frac{11}{13} = \frac{x}{91}$ ________

b $\frac{3}{5} = \frac{42}{x}$ ________

c $\frac{9}{14} = \frac{x}{98}$ ________

4 Simplify.

a $\frac{45}{60}$ = ________

b $\frac{8}{12}$ = ________

5 Multiply.

a $5 \times \frac{2}{3}$ = ________

b $\frac{3}{4} \times 2$ = ________

c $\frac{3}{4} \times \frac{5}{6}$ = ________

6 Divide

a $\frac{1}{4} \div \frac{3}{4}$ = ________

b $6 \div \frac{3}{4}$ = ________

c $\frac{3}{4} \div \frac{2}{7}$ = ________

7 Place the missing numbers.

a $\frac{2}{3} = \frac{1}{12}$ x________

b $\frac{14}{24} = \frac{1}{12}$ x________

c $\frac{7}{8} = \frac{1}{12}$ x________

8 Add the followings:

a $\frac{3}{4} + \frac{7}{12}$ = ________

b $\frac{7}{8} + \frac{3}{4}$ = ________

9 Subtract

a $\frac{4}{5} - \frac{2}{5}$ = ________

b $\frac{5}{6} - \frac{3}{4}$ = ________

10 Simplify

a $2\frac{6}{7} - \frac{3}{8} - \frac{1}{3} - 1\frac{1}{16}$ = ________

b $2\frac{2}{7} - \frac{5}{6} + 8$ = ________

11 Express as improper fractions

a $5\frac{3}{4}$

b $3\frac{5}{64}$

c $1\frac{5}{12}$

Unit, Fractions - Decimal fractions - Addition, subtraction, multiplication & division

Description

Decimal fraction is a fraction whose denominator is 10 or powers of 10 or multiples of 10 (i.e.) 10, 100, 1000, 10000 etc. Meaning of a decimal number:-

12.3256 means

$$(1 \text{ X } 10) + (2 \text{ X } 1) + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$$

Representation

The denominator is omitted. A decimal point is placed at different positions of the number corresponding to the magnitude of the denominator

$$Ex. \frac{5}{10} = 0.5, \frac{35}{100} = 0.35 \; \frac{127}{10000} = 0.0127, \frac{3648}{1000} = 3.648$$

Addition and subtraction

Arrange the decimal fractions in a vertical order, placing the decimal point of each fraction to be added or subtracted, in succession one below the other, so that all the decimal points are arranged in a straight line. Add or subtract as you would do for a whole number and place the decimal point in the answer below the column of decimal points.

Decimal fractions less than 1 are written with a zero before the decimal point. Example: 45/100 = 0.45 (and not simply .45)

Add 0.375 + 3.686

    0.375
    3.686

    4.061

Subtract 18.72 from 22.61

    22.61
    18.72

     3.89

Multiplication

Ignore the decimal points and multiply as whole numbers. Find the total number of digits to the right of the decimal point. Insert the decimal point in the answer such that the number of digits to the right of the decimal point equals to the sum of the digits found to the right of the decimal points in the problem.

Multiply 2.5 by 1.25

= 25 x 125 = 3125. The sum of the figures to the right of decimal point is 3. Hence the answer is 3.125.

Division

Move the decimal point of the divisor to the right to make it a full number. Move the decimal point in the dividend to the same number of places, adding zeroes if necessary. Then divide.

Divide 0.75 by 0.25

$$0.25\overline{)0.75}$$

$$\frac{0.75}{0.25} \text{ x } \frac{100}{100} = \frac{75}{25}$$

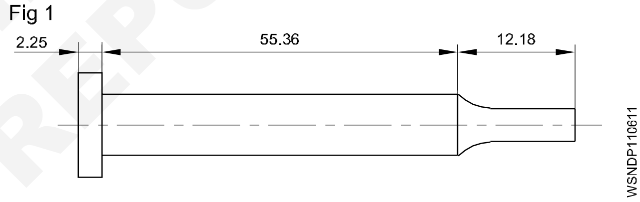
$$25\overline{)75} = 3$$

Move the decimal point in the multiplicand to the right to one place if the multiplier is 10, and to two places if the multiplier is 100 and so on. When dividing by 10 move the decimal point one place to the left, and, if it is by 100, move them point by two places and so on.

Example

Allowance allowing 3 mm for cutting off each pin, how many pins can be made from a 900 mm long bar and how much material will be left out?

Fig 1



Total Length of pin = 2.25 + 55.36 + 12.18

= 69.79 mm

Cutting allowance = 3 mm

Total Length = length of pin + cutting allowance

= 69.79 mm + 3 mm

= 72.79 mm

Length of the bar = 900 mm

No.of pins to be cut $= \frac{900}{72.79} = 12.394$

= 12 pins

Left out material = Total length - length of pin + cutting allowance

= 900 - 12 x 69.79 + 12 x 3

= 900 - 837.48 + 36

= 900 - 873.48

Left out length = 26.52 mm

Conversion of Decimals into fractions and vice-versa

- Convert decimal into fractions

Example

Convert 0.375 to a fraction

Now place 1 under the decimal point followed by as many zeros as there are numbers

$$0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$$

$$0.375 = \frac{3}{8}$$

- Convert fraction into decimal

Example

- Convert $\frac{9}{16}$ to a decimal

Proceed to divide $\frac{9}{16}$ in the normal way of division but put zeros (as required) after the number 9 (Numerator)

```
     0.5625
16 )90000
    80
    100
     96
      40
      32
       80
       80
        0
```

$\frac{9}{16} = 0.5625$

Recurring decimals

While converting from fraction to decimals, some fractions can be divided exactly into a decimal. In some fractions the quotient will not stop. It will continue and keep recurring. These are called recurring decimals.

Examples

- convert $\frac{1}{3}, \frac{2}{3}, \frac{1}{7}$

a $\frac{1}{3} = \frac{10000}{3} = 0.3333 - \text{Recurring}$

b $\frac{2}{3} = \frac{20000}{3} = 0.666 - \text{Recurring}$

c $\left(\frac{1}{7} = \frac{10000}{7} = 0.142857142 - \text{Recurring}\right)$

Method of writing approximations in decimals

1.73556	= 1.7356	Correct to 4 decimal places
5.7343	= 5.734	Correct to 3 decimal places
0.9345	= 0.94	Correct to 2 decimal places

Multiplication and division by 10,100,1000

Multiplying decimals by 10

A decimal fraction can be multiplied by 10,100,1000 and so on by moving the decimal point to the right by as many places as there are zeros in the multiplier.

- 4.645 x 10 = 46.45 (one place)
- 4.645 x 100 = 464.5 (two places)
- 4.645 x 1000 = 4645 (three places)

Dividing decimals by 10

A decimal fraction can be divided by 10,100,1000 and so on, by moving the decimal point to the left by as many places as required in the divisor by putting zeros

Examples

- 3.732 ÷ 10 = 0.3732 (one place)
- 3.732 ÷ 100 = 0.03732 (two places)
- 3.732 ÷ 1000 = 0.003732 (three places)

Examples

- Rewrite the following number as a fraction

  453.273

$$= (4\times100)+(5\times10)+(3\times1)+\frac{2}{10}+\frac{7}{100}+\frac{3}{100}$$

$$= 453\frac{273}{1000}$$

- Write the representation of decimal places in the given number 0.386

  3 - Ist decimal place
  8 - IInd decimal place
  6 - IIIrd decimal place

- Write approximations in the following decimals to 3 places.

  a 6.9453 ——> 6.945
  b 8.7456 ——> 8.746

- Convert fraction to decimal

$$\frac{21}{24} = \frac{7}{8} = 0.875$$

- Convert decimal to fraction

$$0.0625 = \frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

Assignment

1 Write down the following decimal numbers in the expanded form.

a 514.726

b 902.524

2 Write the following decimal numbers from the expansion.

a $500 + 70 + 5 + \frac{3}{10} + \frac{2}{100} + \frac{9}{1000}$

b $200 + 9 + \frac{1}{10} + \frac{3}{100} + \frac{5}{1000}$

3 Convert the following decimals into fractions in the simplest form.

a 0.72

b 5.45

c 3.64

d 2.05

4 Convert the following fraction into decimals

a $\frac{3}{5}$

b $\frac{10}{4}$

c $24\frac{54}{1000}$

d $\frac{12}{25}$

e $\frac{8}{25}$

f $1\frac{3}{25}$

5 Addition of decimals

a 4.56 + 32.075 + 256.6245 + 15.0358

b 462.492 + 725.526 + 309.345 + 626.602

6 Subtract the following decimals

a 612.5200 – 9.6479

b 573.9246 – 215.6000

7 Add and subtract the following

a 56.725 + 48.258 – 32.564

b 16.45 + 124.56 + 62.7 - 3.243

8 Multiply the following

a By 10,100,1000

i 3.754 x 10

ii 8.964 x 100

iii 2.3786 x 1000

iv 0.005 x 1000

b By whole numbers

i 8.4 x 7

ii 56.72 x 8

c By another decimal figure (use calculator)

i 15.64 x 7.68

ii 2.642 x 1.562

9 Divide the following

a $\frac{62.5}{25}$

b $\frac{64.56}{10}$

c $\frac{0.42}{100}$

d $\frac{48.356}{1000}$

10 Division

a $\frac{16.8}{1.2}$

b $\frac{1.54}{1.1}$

11 Change the fraction into a decimal

i $1\frac{5}{8}$

ii $\frac{12}{25}$

12 Find the value

20.5 x 40 ÷ 10.25 + 18.50

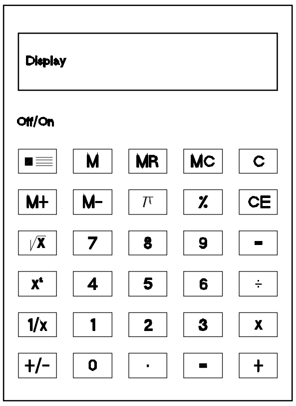
Unit, Fractions - Solving problems by using calculator

A pocket calculator allows to spend less time in doing tedious calculations. A simple pocket calculator enables to do the arithmetical calculations of addition, subtraction, multiplication and division, while a scientific type of calculator can be used for scientific and technical calculations also.

No special training is required to use a calculator. But it is suggested that a careful study of the operation manual of the type of the calculator is essential to become familiar with its capabilities. A calculator does not think and do. It is left to the operator to understand the problem, interpret the information and key it into the calculator correctly.

Constructional Details (Fig 1)

Fig 1



WSNDP110711

The key board is divided into five clear and easily recognizable areas and the display.

- **Data entry keys**

The entry keys are from [0] ....to [9]

and a key for the decimal point [.] .

- **Clearing keys**

These keys have the letter 'C'

[C] [CLR] Clear totally

[CE] Clear entry only

[CM] , [MC] Clear memory

[+] Addition key

[-] Subtraction key

[x] Multiplication key

[÷] Division key

[=] Equals key to display the result

- **Function keys**

[π] Pi key

[$\sqrt{x}$] Square root key

[%] Percentage key

[+/-] Sign change key

[x^2] Square key

[$\frac{1}{X}$] Reciprocal key

- **Memory keys**

[M] Store the display number

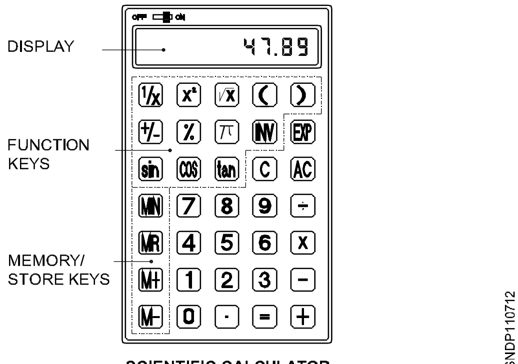
[M+] The displayed value is added to the memory

[M-] The displayed value is subtracted from the memory

[MR] [RCL] The stored value is recalled on to the display

Further functional keys included in Scientific calculators are as shown in Fig 2.

Fig 2



SCIENTIFIC CALCULATOR

[Sin] [Cos] [Tan] [(] [)] For trigonometric functions and for brackets

[Exp] Exponent key

[INV] Some of the keys have coloured lettering above or below them. To use a function in coloured lettering, press INV key. INV will appear on the display. Then press the key that the coloured lettering identifies. INV will disappear from the display.

[log], [INV] [10^x] to obtain the logarithm of the displayed number and the antilogarithm of the displayed value.

[INV] [R–P] to convert displayed rectangular coordinates into polar coordinates.

[INV] [P–R] to convert displayed polar coordinates into rectangular coordinates.

- **The display**

The display shows the input data, interim results and answers to the calculations.

> **The arrangement of the areas can differ from one make to another. Keying in of the numbers is done via. an internationally agreed upon set of ten keys in the order that the numbers are written.**

Rules and Examples:

- **Addition:** Example 18.2 + 5.7

Sequence	Input	Display
Input of the 1st term of the sum	1 8 . 2	18.2
Press + key	+	18.2
Input 2nd term of the sum. the first term goes into the register	5 . 7	5.7
Press the = key	=	23.9

- **Subtraction:** Example 128.8 - 92.9

Sequence	Input	Display
Enter the subtrahend	1 2 8 . 8	128.8
Press - key	-	128.8
Enter the minuend. The subtrahend goes into the register	9 2 . 9	92.9
Press the = key	=	35.9

- **Multiplication:** Example 0.47 x 2.47

Sequence	Input	Display
Enter multiplicand	. 4 7	0.47
Press x key	x	0.47
Enter multiplier, multiplicand goes to register	2 . 4 7	2.47
Press = key	=	1.1609

- **Division:** Example 18.5/2.5

Sequence	Input	Display
Enter the dividend	1 8 . 5	18.5
Press ÷ Key	÷	18.5
Enter the divisor goes to the register	2 . 5	2.5
Press = key	=	7.4

- **Multiplication & Division:**
  Example : 2.5 x 7.2 / 4.8 x 1.25

Sequence	Input	Display
Enter 2.5	2 . 5	2.5
Press x key	x	2.5
Enter 7.2	7 . 2	7.2
Press ÷ key	÷	18
Enter Open bracket	(	
Enter 4.8	4 . 8	4.8
Press x key	x	4.8
Enter 1.25	1 . 2 5	1.25
Enter Close bracket	)	6
Press = key	=	3.0

- Store in memory Example (2+6) (4+3)

Sequence	Input	Display
Workout for the first bracket	2	2
	+	2
	6	6
	=	8
Store the first result in	STO , M	8
x	or M+	
Workout for the 2nd bracket	4	4
	+	4
	3	3
	=	7
Press x key	x	7
Recall memory	RCL or MR	8
Press = key	=	56

- **Percentage:** Example 12% of 1500

Sequence	Input	Display
Enter 1500	1 5 0 0	1500
Press x key	x	1500
Enter 12	1 2	12
Press INV %	INV %	12
Press = key	=	180

- **Square root:** Example $\sqrt{2}+\sqrt{3\times5}$

Sequence	Input	Display
Enter 2	2	2
Press $\sqrt{a}$ key	$\sqrt{a}$	1.414
Press + key	+	1.414
Press bracket key	(	1.414
Enter 3	3	3
Press $\sqrt{a}$ key	$\sqrt{a}$	1.732
Press x key	x	1.732
Enter 5	5	5
Press $\sqrt{a}$ key	$\sqrt{a}$	2.236
Press bracket close key	)	3.873
Press = key	=	5.2871969
2 √ + (3 √ x 5 √) =		5.2871969

$\sqrt{2}+\sqrt{3\times5} = 5.287$

- **Common logarithm:** Example log 1.23

Sequence	Input	Display
1 . 2 3 log	=	0.0899051

- **Power:** Example 123 + 30^2

Sequence	Input	Display
1 2 3 + 3 0 INV X^2	=	1023

- **Before starting the calculations be sure to press the 'ON' key and confirm that '0' is shown on the display.**
- **Do not touch the inside portion of the calculator. Avoid hard knocks and unduly hard pressing of the keys.**
- **Maintain and use the calculator in between the two extreme temperatures of 0° and 40° C.**

- **Never use volatile fluids such as lacquer, thinner, benzine while cleaning the unit.**
- **Take special care not to damage the unit by bending or dropping.**
- **Do not carry the calculator in your hip pocket.**

Assignment

1 Using calculator solve the following

a 625 + 3467 + 20 + 341 + 6278 = ______

b 367.4 + 805 + 0.7 + 7.86 + 13.49 = ______

c 0.043 + 1.065 + 13.0 + 34.76 + 42.1 = ______

d 47160 + 1368.4 + 0.1 + 1.6901 + 134.267 = ______

2 Using calculator simplify the following

a 24367 - 4385 = ______

b 9.643 - 0.7983 = ______

c 4382.01 - 381.3401 = ______

d 693.42 - 0.0254 = ______

3 Using calculator find the values of the following

a 23 x 87 = ______

b 1376 x 0.81 = ______

c 678 x 243 = ______

d 0.75 x 0.24 = ______

4 Using calculator solve the following

a 22434 ÷ 3 = ______

b 4131 ÷ 243 = ______

c 469890 ÷ 230 = ______

d 3.026 ÷ 0.89 = ______

5 Solve the following

a $\dfrac{1170 \times 537.5}{13 \times 215}$ = ______

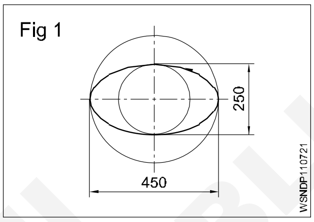
b $\dfrac{28.2 \times 18 \times 3500}{1000 \times 3 \times 0.8}$ = ______

6 Solve the following

a $\dfrac{(634+128) \times (384 - 0.52)}{8 \times 0.3}$ = ______

b $\dfrac{(389 - 12.2) \times (842 - 0.05 - 2.6)}{(3.89 - 0.021) \times (28.1 + 17.04)}$ = ______

7



2a = 450 mm(major axis)

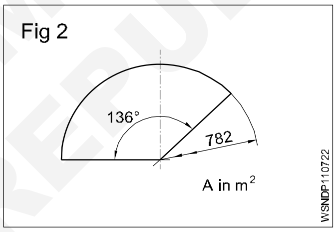
2b = 250mm(minor axis)

Perimeter of the ellipse

A = ______ $metre^2$

Hint A = π x a x b $unit^2$

8



ø = 782 mm

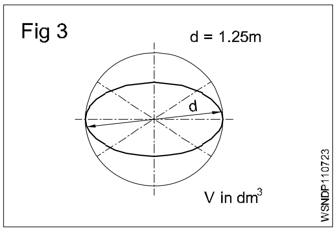
α = 136°

Area of the sector

A = ______

Hint $A = \dfrac{\pi \times d^2}{4} \times \dfrac{\alpha}{360°}$

9



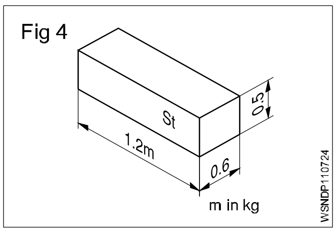
d = 1.25 metre

V = ______ dm^3

Volume of sphere

Hint $V = \dfrac{4}{3} \pi r^3$

10



L = 1.2 metres

B = 0.6 metre

H = 0.5 metre

'ρ' (rho) density of steel

= 7.85 kg/dm^3'

m = ______ kg

(mass 'm = V x ρ)

Square root, Ratio and Proportions, Percentage - Square and square root

a basic number
2 exponent

$\sqrt{\ }$ radial sign indicating the square root.

$\sqrt{a^2}$ square root of 'a' square

a² radicand

Square number

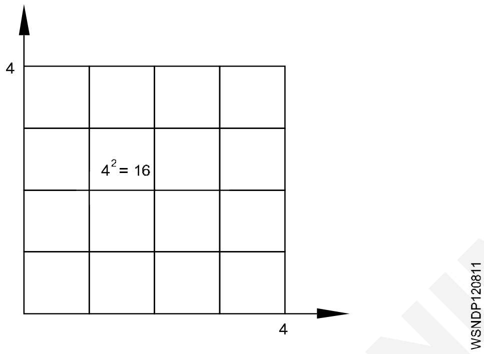
The square of a number is the number multiplied by itself.

Basic number x basic number = Square number

$a \times a = a^2$

$4 \times 4 = 4^2 = 16$

Fig 1



Splitting up

A square area can be split up into sub-areas. The largest square of 36 is made up of a large square 16, a small square 4 and two rectangles 8 each.

Large square 4 x 4 = 16 a^2

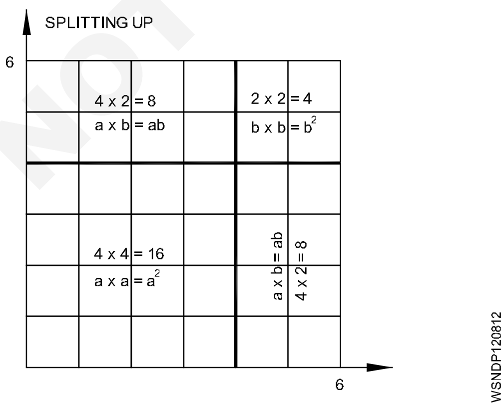
Two rectangles 2 x 4 x 2 = 16 2ab

Small square 2 x 2 = 4 b^2

Sum of sub-areas = 36 = $a^2 + 2ab + b^2$

$$\sqrt{36} = \sqrt{a^2 + 2ab + b^2}$$

Fig 2

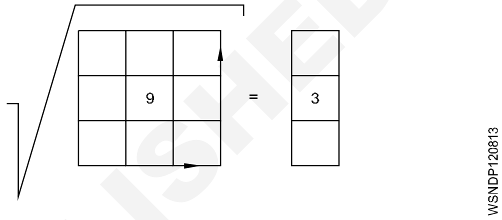


Result: In order to find the square root, we split up the square numbers.

Extracting the square root procedure

- Starting from the decimal point form groups of two figures towards right and left. Indicate by a prime symbol. $\sqrt{4624.00}$
- Find the root of the first group, calculate the difference, bring down the next group.
- Multiply the root by 2 and divide the partial radicand.
- Enter the number thus calculated in the divisor for the multiplication.

Fig 3



If there is a remainder, repeat the procedure.

```
        68
   6 | 46,24
       36
 128 | 1024
       1024
          0
```

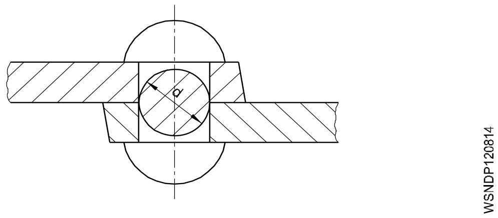
$\sqrt{4624} = 68$

> **Basic number x basic number = Square**
>
> $\sqrt{\textbf{Square number}} = \textbf{basic number}$

Example

The cross-section of a rivet is 3.46 cm². Calculate the diameter of the hole.

Fig 4



Rivet cross-section is the hole cross-section.

To find 'd',

Given that Area = 3.46 cm²

Area = 0.785 x d² (formula)

3.46 cm² = d² x 0.785

$$d^2 = \frac{3.46\ cm^2}{0.785}$$

$$d = \sqrt{\frac{3.46}{0.785}}\ cm$$

$$d = 2.1\,cm\ (or)\ 21mm$$

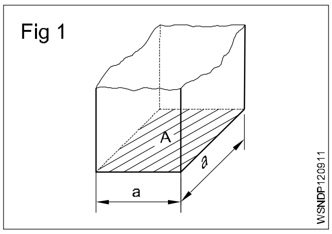
Square root, Ratio and Proportions, Percentage - Simple problems using calculator

1 a $\sqrt{2916}$ = ______________.

b $\sqrt{45796}$ = ______________.

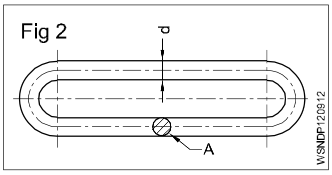
c $\sqrt{8.2944}$ = ______________.

d $\sqrt{63.845}$ = ______________.

2 

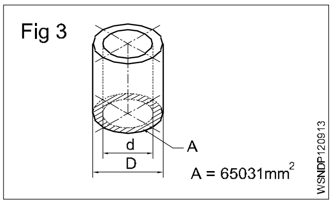
A = 2025 mm^2

a = ____________mm

3 

A = 176.715 mm^2

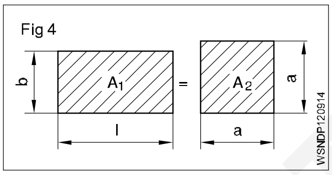
d = ____________mm

4 

A = 65031 mm^2

d = 140 mm

D = ____________mm

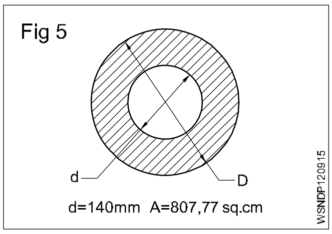
5 

l = 58 cm

b = 45 cm

$A_1 = A_2$

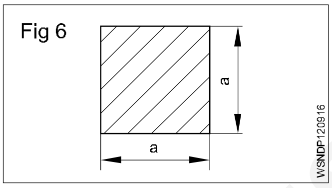
a = ____________cm

6 

A = 807.77 cm^2

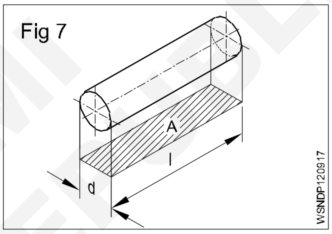
d = 140 mm

D = ____________mm

7 

a x a = 543169 mm^2

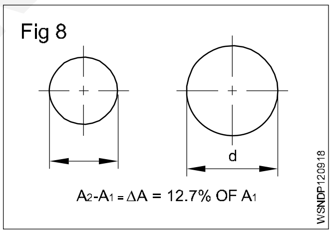
a = ____________mm

8 

d : l = 1:1.5

A = 73.5 mm^2

d = ____________mm

9 

increase in area

A = 12.7%

A = 360 mm^2

d = ____________mm

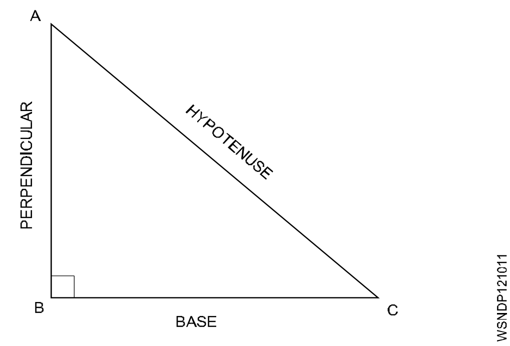
(d = diameter after the increase in area)

Square root, Ratio and Proportions, Percentage - Applications of pythagoras theorem and related problems

Applications of Pythagoras Theorem

Some of the applications of the Pythagoras theorem are; (Fig 1)

Fig 1



1 The Pythagoras theorem is commonly used to find the lengths of sides of a right-angled triangle.

2 It is used to find the length of the diagonal of a square.

3 Pythagoras theorem is used in trigonometry to find the trigonometric ratios like sin, cos, tan, cosec, sec and cot.

4 Pythagoras theorem is used in security cameras for face recognition.

5 Architects use the technique of the Pythagoras theorem for engineering and construction fields.

6 The Pythagoras theorem is applied in surveying the mountains.

7 It is also used in navigation to find the shortest route.

8 By using the Pythagoras theorem, we can derive the formula for base, perpendicular and hypotenuse.

9 Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.

10 Pythagoras theorem is used to calculate the steepness of slopes of hills or mountains.

11 The converse of the Pythagoras theorem is used to check whether a triangle is a right triangle or not.

Application of pythagoras theorem in real life

Pythagoras theorem states that

"In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides".

1 The sides of this triangle have been named Perpendicular, Base and Hypotenuse.

2 The hypotenuse is the longest side, as it is opposite to the angle 90°.

3 The sides of a right triangle (say AB, BC and CA) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triplet.

4 To calculate the length of staircase required to reach a window

5 To find the length of the longest item can be kept in your room.

6 To find the steepness of the hills or mountains.

7 To find the original height of a tree broken due to heavy rain and lying on itself

8 To determine heights and measurements in the construction sites.

Examples

1 What is the side AC if AB = 15 cm, BC = 25 cm.

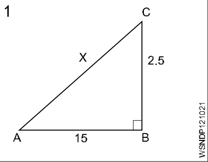
$AC^2 = AB^2 + BC^2$

$= 15^2 + 25^2$

$= 225 + 625 = 850$

$AC = \sqrt{850} = 29.155$ cm

Fig 1



2 What is the side BC if AB = 10 cm, AC = 30 cm.

$AC^2 = AB^2 + BC^2$

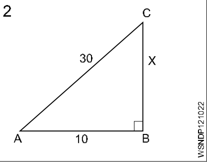
$30^2 = 10^2 + BC^2$

$900 = 100 + BC^2$

$BC^2 = 900 - 100 = 800$

$BC = 28.284$ cm

Fig 2



3 What is the side AB if BC = 20 cm, AC = 35 cm.

$AC^2 = AB^2 + BC^2$

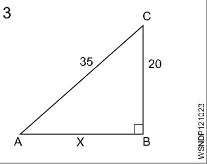
$35^2 = AB^2 + 20^2$

$1225 = AB + 400$

$AB^2 = 1225 - 400 = 825$

$AB = 28.72$ cm

Fig 3



4 What is the value of side BC if AB = 8 cm, AC = 24 cm.

$AC^2 = AB^2 + BC^2$

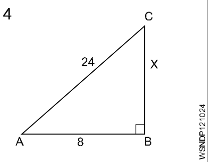
$24^2 = 8^2 + BC^2$

$576 = 64 + BC^2$

$BC^2 = 576 - 64 = 512$

$BC = \sqrt{572} = 22.63$ cm

Fig 4



5 What is the value side AC if AB = 6.45 cm, BC = 8.52 cm.

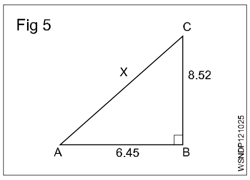
$AC^2 = AB^2 + BC^2$

$AC^2 = 6.45^2 + 8.52^2$

$AC^2 = 41.60 + 72.59$

$= 114.19$

$AC = \sqrt{114.19} = 10.69$ cm



6 What is the value of side AB if BC = 3.26 cm, AC = 8.24 cm.

$AC^2 = AB^2 + BC^2$

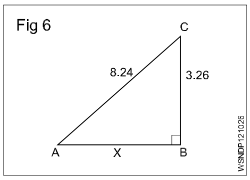
$8.24^2 = AB^2 + 3.26^2$

$67.9 = AB + 10.63$

$AB^2 = 67.9 - 10.63$

$= 57.27$

$AB = \sqrt{57.27} = 7.57$ cm



7 What is the value of side AB if AC = 12.5 cm, BC = 8.5 cm.

$AC^2 = AB^2 + BC^2$

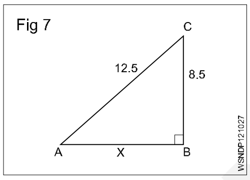
$12.5^2 = AB^2 + 8.5^2$

$156.25 = AB + 72.25$

$AB^2 = 156.25 - 72.25$

$= 84$

$AB = \sqrt{84} = 9.17$ cm



8 A ladder of 12.5 metre long is placed with upper end against a wall. The lower end being 7.5 metres from the wall. What height is the upper end above the ground.

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$BC^2 = x^2$

$AC^2 = AB^2 + BC^2$

$12.5^2 = x^2 + 7.5^2$

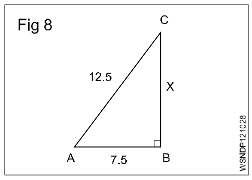
$x^2 = (12.5)^2 - (7.5)^2$

$= (12.5 + 7.5)(12.5 - 7.5)^2$

$= 20 \times 5$

$= \sqrt{100} = 10$

$x = 10$ m



9 What is the value of AB.

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

$AB^2 = x^2$

$AC^2 = AB^2 + BC^2$

$10^2 = x^2 + 6^2$

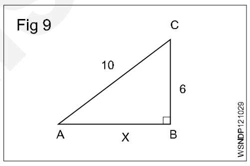
$x^2 = 10^2 - 6^2$

$= 100 - 36 = 64$

$x = \sqrt{64}$

$= 8$

$AB = 8$



Assignment

1 What is the value of side AB, in a right angled triangle of side AC = 10 cm and BC = 5 cm.

2 What is the value of side AC, in a right angled triangle of side AB = 6.5 cm and BC = 4.5 cm.

3 What is the value of side BC, in a right angled triangle of side AC = 14.5 cm and AB = 10.5 cm.

4 What is the value of side AC, in a right angled triangle of side AB = 7 cm and BC = 5 cm.

5 What is the value of side BC, in a right angled triangle of side AC = 13.25 cm and AB = 8.75 cm.

Square root, Ratio and Proportions, Percentage - Ratio and proportion

Ratio

Introduction

It is the relation between two quantities of the same kind and is expressed as a fraction.

Expression

a, b two quantities of the same kind. $\frac{a}{b}$ or a:b or a $\div$ b or a in b is the ratio.

Ratio is always reduced to the lowest terms.

Example

$$7:14 = \frac{7}{14} = \frac{1}{2} = 1:2$$

Proportion

It is the equality between the ratios, a : b is a ratio and c: d is another ratio. Both ratios are equal. Then

a :b :: c : d or $\frac{a}{b} = \frac{c}{d}$

Example

250 : 2000 :: 1 : 8

Proportion fundamentals

If $\frac{a}{b} = \frac{c}{d}$ then

- ad = bc
- $\frac{a}{c} = \frac{b}{d}$
- $\frac{b}{a} = \frac{d}{c}$
- $\frac{a+b}{b} = \frac{c+d}{c}$ and $\frac{a+b}{a} = \frac{c+d}{c}$
- $\frac{a-b}{b} = \frac{c-d}{d}$
- $\frac{a+b}{b+d} = \frac{a}{c} = \frac{c}{d}$

3:4::6:8 or $\frac{3}{4} = \frac{6}{8}$

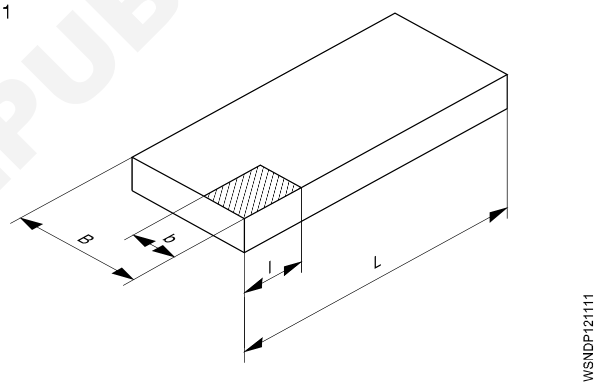
- 3 x 8 = 6 x 4
- $\frac{3}{6} = \frac{4}{8}$
- $\frac{4}{3} = \frac{8}{6}$
- $\frac{3+4}{4} = \frac{6+8}{8}$
- $\frac{3-4}{4} = \frac{6-8}{8}$
- $\frac{3+6}{4+8} = \frac{9}{12} = \frac{3}{4}$

Ratio - relation of two quantities of the same kind.
Proportion - equality between two ratios.

Example

- A steel plate of 800 x 1400 mm is to be drawn to a scale of 1:20. What will be the lengths in the Fig 1.

Fig 1



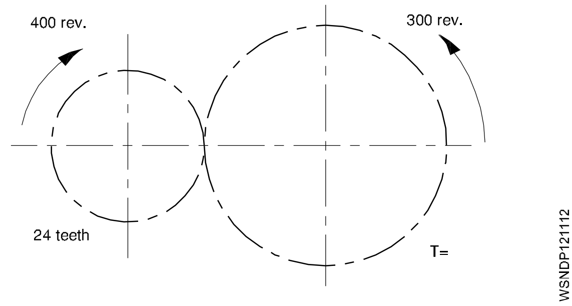
The reduction ratio is $\frac{1}{20}$.

B is reduced from 800 to 800 x $\frac{1}{20}$ = 40 mm.

L is reduced from 1400 x $\frac{1}{20}$ = 70 mm.

- Find the number of teeth of the larger gear in the gear transmission shown in the Fig 2.

Fig 2



Speed ratio = 400 : 300

Teeth ratio = 24:T

$$\frac{400}{300} = \frac{T}{24}$$

$$\therefore T = \frac{24 \times 400}{300} = 32 \text{ Teeth}$$

Find the ratio of A:B:C

If A:B= 2:3 and B:C=4:5

A:B = 2:3

B:C = 4:5

A:B = 8 :12 (Ratio 2:3 multiply by 4)

B:C = 12:15 (Ratio 4:5 multiply by 3)

∴ A:B:C = 8:12:15

Assignment

1

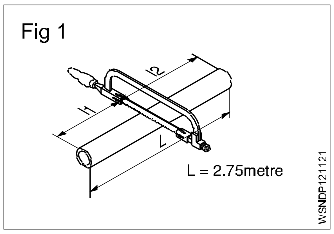


Fig 1

$l_1 : l_2 = 2:3$

L = 2.75 metres

l_1 = ______ metres

l_2 = ______ metres

2

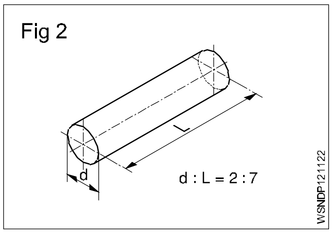


Fig 2

d: L of shaft = 2:7

d = 40 mm

L = ______ mm

3

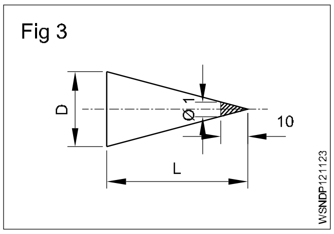


Fig 3

D : L=1:10

L=150mm

D=______ mm

4

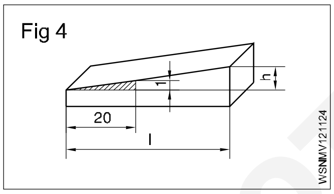


Fig 4

$$\frac{\Delta h}{l} = \frac{1}{20}$$

l = 140 mm

Δh = _____ mm

5

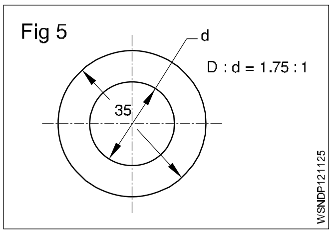


Fig 5

D : d = 1.75: 1

D = 35 mm

d = ______ mm

6

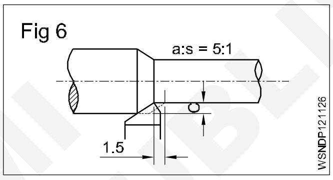


Fig 6

a:s = 5:1

s = 1.5mm

a =______ mm

7 A:B=9:12

B:C=8:10

Then A:B:C=___________

8 A:B=5:6

B:C=3:4

Then A:B:C=______

9 A:55=9:11

A = __________

10 15:9.3=40:x

x = __________

Square root, Ratio and Proportions, Percentage - Ratio and Proportions - Direct and indirect proportions

Proportion

Description

It is the equality between the ratios, a:b is a ratio and c:d is another ratio. Both ratios are equal. Then

a : b::c : d or e.g. 250 : 2000::1 : 8

Rule of three

A three step calculation

statement

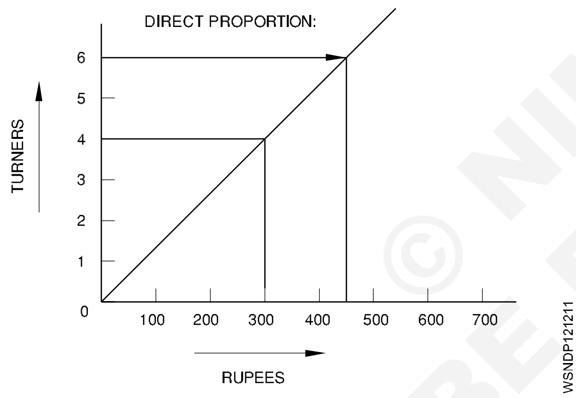
single

multiple.

Direct proportion

The more in one the more in the other - An increase in one denomination produces an increase in the other. (Fig 1)

Fig 1



Examples

1 4 turners earn 300 Rupees. How much will 6 Turners earn?

Statement
4 turners = 300 Rupees

Single
1 Turner = 75 Rupees

Multiple
6 Turners = 6 x 75 = 450 Rupees

2 One vehicle consumes 30 litres of petrol per day how much petrol is used by 6 Vehicles operating under similar condition.

One vehicle uses petrol = 30 litres per day.

Then six vehicles will use = 6 Times as much

= 6 x 30 = 180 litres/day.

3 4 vehicles consumes 120 gallons of petrol per day how much petrol will be used by 12 vehicles operating under the same condition.

4 vehicles use 120 gallons per day

1 Vehicle will use $\frac{120}{4}$ = 30 gallons/day

12 vehicles will use 12 x 30 = 360 gallons/day

Both examples are called simple proportion because only two quantities were used and the day is common for both ratios.

4 If 2 litres of petrol costs Rs 60. Find the cost of 50 litres.

Quantity of Petrol	Cost of Petrol
2 litres	Rs.60
50 litres	x

1 litre petrol $= \frac{60}{2}$ = Rs.30

50 litres petrol = 30 x 50 = Rs 1500

5 A 150mm dia gear meshes with 50mm dia gear. If the larger gear has 30 teeth. How many teeth will have the smaller gear have?

Gear dia	No. of Teeth
150 mm	30
50 mm	x

$x = \frac{50}{150} \times 30$ = 10 teeth.

6 A mechanic assembles 7 machines in 2½ days. How long will it take time to assemble 70 machines at the same rate.

Machines	Days
7	2½
70	x

$x = \frac{70 \times 2.5}{7}$ = 25 days

Assemble for 70 machines will take 25 days.

7 A roll of wire weighs 1.24 kg from this roll a piece of 3.7cm long is cut and it is found to weigh 2.93 gm. What is the length of the wire in the roll?

Weight of wire	Length of wire
2.93 gm	3.7 cm
1.24 kg (1240 gm)	x

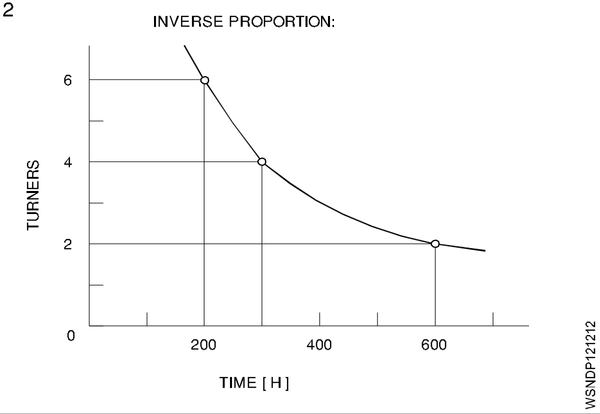
$$x = \frac{1240}{2.93} \times 3.7 = 1566 \text{ cm}$$

Length of wire = 1566 cm.

Indirect or inverse proportion

The more in one the lesser other - Increase in one quantity will produce a decrease in the other. (Fig 2)

Fig 2



Example

1 4 turners finish a job in 300 hours. How much time will 6 turners take to do the same job?

Solution procedure in three steps:

Statement 4 turners taken = 300 hours

The time will reduce if 6 turners to do the same job. Therefore this is inverse proportion.

Multiple fraction $\frac{4 \text{ Turners}}{6 \text{ Turners}} \times 300$ hours

6 Turners = 200 hours

Result - The more the less.

2 8 workman take 6 days to complete a job. How many days it will take for 4 workman to complete the same job?

Workman	Days
8	6
4	x

$$x = \frac{8}{4} \times 6 = 12 \text{ days}$$

4 workers complete the work = 12 days.

3 5 men working on a job finished it in 32 days. Find out in how many days 8 men will finish the same job?

Men	Days
5	32
8	x

$$x = \frac{5 \times 32}{8} = 4 \times 5 = 20 \text{ days}$$

8 men will complete the job = 20 days.

4 An engine running at 150 rpm drives a shaft by pulley diameter is 55cm and that of the driven shaft pulley is 33 cm. Find the speed of the shaft?

Dia of pulley	Rpm of shaft
55 cm	150
33 cm	x

$$x = \frac{55 \times 150}{33} = 250 \text{ rpm.}$$

Speed of the 33cm diameter will run 250 rpm.

5 A pulley of 80 cm diameter is rotating at 100 rpm and drives another pulley of 40 cm diameter. Find the rpm of driven pulley. If slip is 2.5% find the rpm?

Dia of pulley	Rpm of pulley
80 cm	100
40 cm	x

40 cm diameter = 200 rpm.

Slip is 2.5% = 195 rpm.

Problems involving both

Example

2 turners need 3 days to produce 20 pieces. How long will it take for 6 turners to produce 30 such pieces?

Statement

2 turners, 20 pieces = 3 days

6 turners, 30 pieces = how many days.

First step (Fig 3)

Statement 2 turners for 20 pieces = 3 days

1 turner for 20 pieces = 3 x 2 = 6 days

Multiple 6 turners for 20 pieces = $\frac{6}{6}$ = 1 day

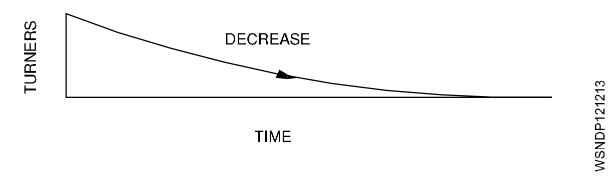
Statement 6 turners for 20 pieces = 1 day

Single 6 turners for 1 piece = $\frac{1}{20}$ days

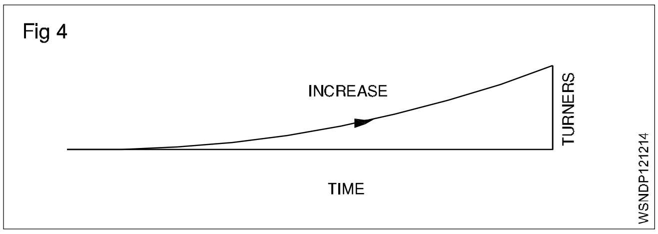
Multiple 6 turners for 30 pieces = $\frac{1}{20} \times 30$ = 1.5 days

Inverse proportion - More the less.

Fig 3



Second step (Fig 4)



Direct proportion - More the more.

Solve the problem by first writing the statement and proceed to single and then to the multiple according to the type of proportion that is involved.

Introduction

Proportional fundamentals, as applicable to motor vehicle calculations are discussed below.

Simple Proportion

- Proportion

This is an equality between two ratios

Compound and Inverse proportions

- Compound proportions

Example

5 Fitter take 21 days to complete overhauling of 6 vehicles how long 7 Fitters will take to over haul 8 vehicles (Assume time of overhauling each vehicle is constant)

In this both direct and indirect proportions are used.

- 1 Fitter will overhauling 1 vehicle in days (shorter time).
- Quantities (No. of days) are taken in last as that is the answer required in this case.

Fitters	Vehicle	Days
5	6	21
7	8	x

$$\left(\frac{21\times5}{6\times7}\times8\right)=20 \text{ days}$$

Ans: 7 Fitters will overhaul 8 vehicles in 20 days.

Inverse proportion

Some times proportions are taken inversely.

Examples

- If one water pump fills the fuel tank in 12 minutes, two pumps will take half the time taken.

The time should not be doubled.

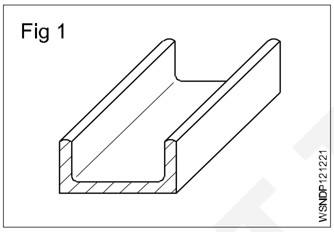
- 2 pumps will take 30 minutes to fill up a tank how long will 6 similar pumps take this to fill this tank.

Pump	Time
2	30
6	x

Ans: Time taken by 6 pumps = $\frac{30\times2}{6}=10$ minutes

Assignment

1

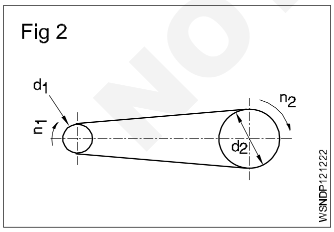


Length = 6.1 metre

Weight = 32 kgf

Weight of 1 metre of the same channel = _______ kgf

2



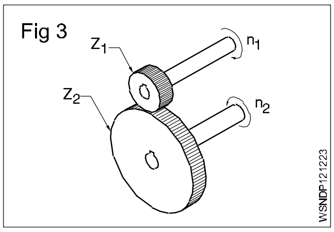
d_1 = 120 mm

d_2 = 720 mm

n_1 = 1200 rpm

n_2 = ______ rpm

3



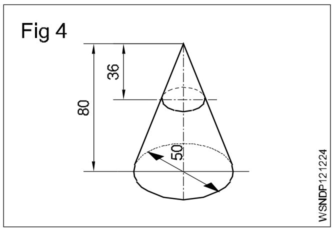
Z_1 = 42 T

n_2 = 96 rpm

n_1 = 224 rpm

Z_2 = _______ T

4



D = 50 mm

H = 80 mm

h = 36 mm

d = ______ mm

5 If a mechanic assembles 8 machines in 3 days, how long he will take to assemble 60 machines.

6 In an auto shop the grinding wheel makes 1000 rpm and the driven pulley is 200 mm dia. If the driving pulley is 150 mm dia. Find out the rpm of the driving pulley.

7 In a gearing of a vehicle the following facts are found.

A 180 mm dia of gear meshes with 60mm dia gear. If the bigger gear makes 60 rpm. What will be the rpm of smaller gear.

8 A vehicular job is completed by 5 mechanics in 4 days. If only 3 mechanics are available, in how many days the work can be completed.

Square root, Ratio and Proportions, Percentage - Percentage

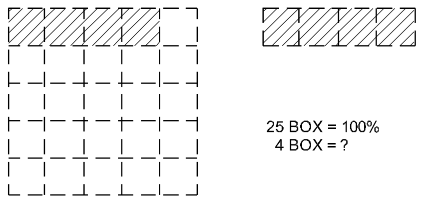
Percentage

Percentage is a kind of fraction whose denominator is always 100. The symbol for percent is %, written after the number. e.g. 16%.

Ex. $\frac{16}{100} = 0.16$

In decimal form, it is 0.16. Percentage calculation also involves rule of three. The statement (the given data), for unit, and then to multiple which is for calculating the answer. (Fig 1)

Fig 1



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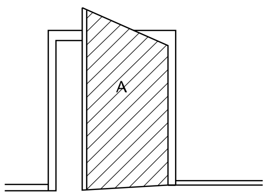
Example

The amount of total raw sheet metal to make a door was 3.6 metre^2 and wastage was 0.18 metre^2. Calculate the % of wastage. (Fig 2)

Fig 2



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Solution procedure in three steps.

Statement:

Area of door (A) =3.6 m^2 = 100 %.

Wastage = 0.18 m^2

Single: $\frac{100}{3.6}$ x 1 m^2

Multiple: for 0.18 m^2= $\frac{100}{3.6}$ x 0.18.

Wastage = 5%.

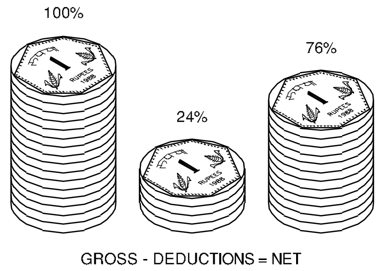
Analyse the given data and proceed to arrive at the answer through the unit.

Example

A fitter receives a take-home salary of 984.50 rupees.

If the deduction amounts to 24%, what is his total salary? (Fig 3)

Fig 3



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Total pay 100%

Deduction 24%

Take home salary 76%

If the take home pay is Rs.76, his salary is 100.

For 1% it is $\frac{1}{76}$

For Rs.984.50, it is $\frac{1}{76}$ x 984.50.

For 100% it is $\frac{984.50}{76}$X100 $=$ 1295.39

100% i.e. gross pay = Rs.1295.40.

Example 1

75 litres of oil is taken out from a oil barrel of 200 litres capacity. Find out the percentage taken in this.

Solution

% of oil taken = Oil taken out (litres) / Capacity of Barrel (litres) x 100

$$= \frac{75}{200} \times 100 = 37\tfrac{1}{2}\%$$

Example 2

A spare part is sold with 15%. Profit to a customer, to a price of Rs.15000/-. Find out the following (a) What is the purchase price (b) What is the profit.

Solution: CP = x ,

CP = cost price

SP = sale price

SP=CP+15%of CP

$$15000 = x + \frac{15x}{100} = \frac{100x + 15x}{100}$$

$$x = \frac{1500000}{115} = 13043.47$$

Profit = SP-CP = 15000-13043.47 = 1956.53

Purchase price = Rs.13,043/,Profit = Rs. 1957

Example 3

Out of 80000 cars, which were tested on road, only 16000 cars had no fault. What is the percentage in this acceptance.

$$= \frac{16000}{80000} \times 100 = \frac{100}{5} = 20\%$$

Example 4

The price of a motor cycle dropped to 92% of original price and now sold at Rs.18000/- What was the original price.

Solution

Present price of Motor cycle Rs.18000

This is the value of 92% of original price

$$\text{Original Price} = 18000 \times \frac{100}{92} = \frac{1800000}{92}$$

= Rs.19565

Example 5

A Motor vehicle uses 100 litres of Petrol per day when travelling at 30 kmph. After top overhauling the consumption falls to 90 litres per day. Calculate percentage of saving.

Solution

Percentage of saving = Decrease in consumption/Original consumption x 100

$$= (100 - 90)\frac{\text{litres}}{100} \times 100$$

$$= \frac{10}{100} \times 100$$

= 10% Saving in fuel.

Assignment

1

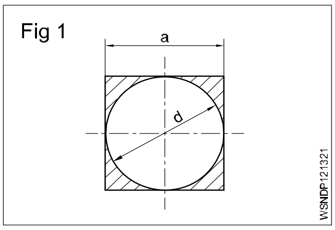


Fig 1

a = 400mm (side of square)

d = 400 mm

Wastage = ______ %.

2

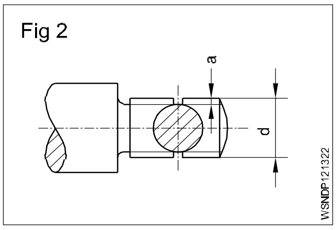


Fig 2

d = 26mm

'a' depth of u/cut = 2.4mm

reduction of area at cross-section

= ______ %

3

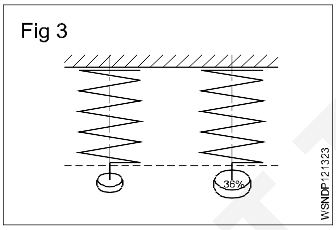


Fig 3

Percentage of increase = 36%

Value of increase = 611.2 N/mm²

Original tensile strength = ______ N/mm².

4

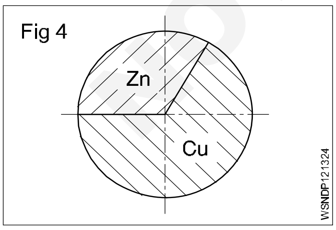


Fig 4

Copper in alloy = 27 kg

Zinc in alloy = 18 kg

% of Copper = ______ %

% of Zinc = ______ %.

5

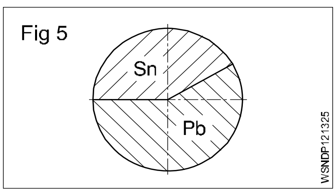


Fig 5

Weight of alloy = 140 Kgf

Weight of Sn 40%

Pb = ______ Kgf

Sn = ______ Kgf.

6

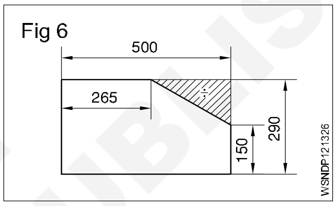


Fig 6

Shaded portion = ______ %.

7

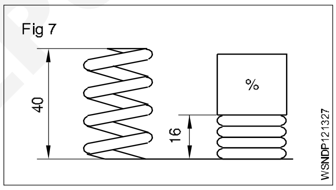


Fig 7

Compression length = ______ %.

8

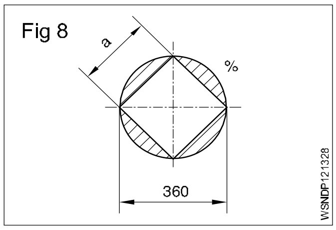


Fig 8

d = 360 mm

a = 0.707 x d

Wastage = ______ %.

9

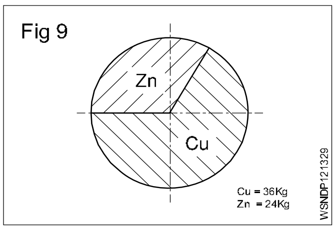


Fig 9

Cu = 36 Kg

Zn = 24 Kg

Cu = ______ %

Zn = ______ %

10

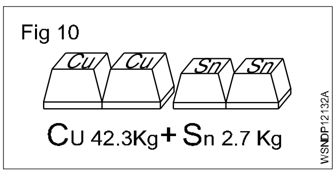


Fig 10

Cu = 42.3 Kg

Sn = 2.7 Kg

Cu ______ %

Sn = ______ %.

Square root, Ratio and Proportions, Percentage - Changing percentage to decimal and fraction

Conversion of Fraction into Percentage

1 Convert $\frac{1}{2}$ into percentage.

Solution: $\frac{1}{2}$ x 100

= 50%

2 Convert $\frac{1}{11}$ into percentage

Solution: $\frac{1}{11}$ x 100 = $\frac{100}{11}$

= 9.01%

Convert the following fraction into percentage.

1 $\frac{1}{4}$

2 $\frac{1}{5}$

3 $\frac{2}{3}$

4 $\frac{3}{8}$

Conversion of Percentage into Fraction

1 Convert 24% into fraction.

Solution: $\frac{24}{100} = \frac{6}{25}$

2 Convert $33\frac{1}{3}$% into fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$

Convert the following percentage into fraction

1 15%

2 $87\frac{1}{2}$%

3 80%

4 12.5%

Conversion of Decimal Fraction into Percentage

1 Convert 0.35 into percentage.

Solution: 0.35 x 100

= 35%

2 Convert 0.375 into percentage.

Solution: 0.375 x 100

= 37.5%

Convert the following Decimal Fraction into Percentage

1 0.2

2 0.004

3 0.875

4 0.052

Conversion of Percentage into Decimal fraction

1 Convert 30% into decimal fraction.

Solution: $\frac{30}{100}$ = 0.3

2 Convert $33\frac{1}{3}$% into decimal fraction.

Solution: $\frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100}$

$= \frac{1}{3}$ = 0.333

Convert the following percentage into decimal fraction

1 15%

2 7%

3 $12\frac{1}{2}$%

4 90%

Mensuration - Area and perimeter of square, rectangle and parallelogram

In Engineering field, an Engineer has to estimate the material, manpower, machinery, etc. required to prepare the geometrical objects. Hence we must be very conversant with all relevant formulae connected with geometrical objects.

Length - l unit

Breadth or width - b unit

Diagonal - d unit

Diameter - d unit

Radius - r unit

Semi perimeter - S unit

Perimeter - P unit

Circumference - C unit

Area - A unit^2

Total surface area - T.S.A unit^2

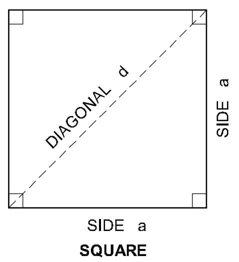
Lateral surface area - L.S.A unit^2

Volume - V unit^3

Square

This is also a four sided figure, opposite sides are parallel. All the four sides are equal. Angle between adjustment side are 90º.

Fig 1



$A = a^2$ (or) unit^2

$P = 4a$ unit

$d = \sqrt{2}\,a$ unit

$a = \frac{d}{\sqrt{2}}$ unit where $\sqrt{2} = 1.414$

Find the area of a brass sheet in the form of a square whose perimeter is 31.2 cm.

Perimeter(P) = 4a = 31.2 cm

$\therefore a = \frac{31.2}{4} = 7.8$ cm

Area (A) $= a^2$

$= 7.8 \times 7.8 = 60.84$ cm^2

Examples

1 Find out the circumference, diagonal and area of a square, whose side is 18 cm.

Side of the square (a) = 18 cm

Perimeter (P) = 4a

= 4 x 18 = 72 cm

Diagonal (d) $= \sqrt{2} \times a$

$= \sqrt{2} \times 18 = 1.414 \times 18$

= 25.45 cm

Area (A) $= a^2$

$= 18 \times 18 = 324$ cm^2

Perimeter of square = 72 cm

Diagonal = 25.45 cm ; Area = 324 cm^2

2 If the diagonal of a square measure 10 cm. Find area of the square.

Diagonal of the square (d) $= \sqrt{2}\,a = 10$ cm

Side (a) $= \frac{d}{\sqrt{2}}$

Area (a^2) $= \frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}} = \frac{d^2}{2}$

$= \frac{10^2}{2} = \frac{100}{2}$

= 50 cm^2

Area of the square = 50 cm^2

3 The perimeter of one square is 748 cm and that of Another is 336 cm. Find the total area of the two squares.

Side of the square (a) $= \frac{\text{Perimeter}}{4}$

1st square

Side (a) $= \frac{\text{Perimeter of } 1^{st} \text{ square}}{4}$

$= \frac{748}{4} = 187$ cm

Area (A) $= a^2$

= 187 x 187

= 34,969 cm^2

2nd square

Side (a) $= \frac{\text{Perimeter of } 2^{nd} \text{ square}}{4}$

$= \frac{336}{4} = 84$ cm

Area (A) $= a^2$

$= 84 \times 84$

$= 7{,}056 \text{ cm}^2$

Total area of two squares = 34,969 + 7,056

$= 42{,}025 \text{ cm}^2$

Assignment

1 Find the Area, Perimeter and diagonal of a square steel plate whose side measures 28.1 cm.

2 Find the area of a square whose diagonal is equal to 8.5 cm.

3 Find the area of the square if the side of the square is 28 cm.

4 Find its side if the area of the square field is 169 m^2.

5 Find the area of the square if the diagonal of the square is 20 cm.

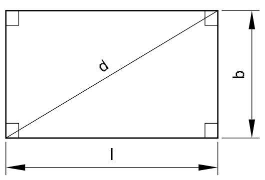
6 Find the perimeter of a square whose diagonal is 144 m.

7 Find the area if the perimeter of a square plot is 48 m.

Rectangle

This is a four sided figure. Opposite sides are parallel. Angles between adjacent sides are 90°.

Fig 1



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A = Area = length x breadth = $l.b.\text{unit}^2$

P = Perimeter = 2 (l + b) unit

Diagonal = $\sqrt{l^2 + b^2}$ unit

Examples

1 Find the Area, Perimeter and diagonal of a rectangle whose length and breadth are 144 mm and 60 mm respectively.

Area = A $= l \times b \text{ unit}^2$

$= 144 \times 60 = 8640 \text{ mm}^2$

Perimeter = P = 2 (l + b) unit

= 2(144 + 60)

= 2 x 204 = 408 mm

Diagonal = d $= \sqrt{l^2 + b^2}$ unit

$= \sqrt{144^2 + 60^2}$

$= \sqrt{20736 + 3600}$

$= \sqrt{24336} = 156$ mm

2 The perimeter of a rectangle is equal to 42 cm. If its breadth is 9 cm. Find the length of the rectangle.

Perimeter = 42 cm

Breath = 9 cm

Length = ?

Perimeter = P = 2(l + b)

2(l + 9) = 42

l + 9 = 42 ÷ 2

l + 9 = 21

l = 21 - 9

l = 12 cm

3 The perimeter of a rectangle is 48 cm and its length is 4 cm more than its width. Find the length and breadth of the rectangle.

Perimeter (P) = 48 cm

Breath (b) = x

Length (l) = $x + 4$

2(l + b) = Perimeter

$2(x + 4 + x) = 48$

$2(2x + 4) = 48$

$4x + 8 = 48$

$4x = 48 - 8$

$x = \frac{40}{4} = 10$

x = breadth = 10 cm

length $= x + 4 = 10 + 4 =$ **14 cm**

4 How many rectangular pieces of 50 cm x 20 cm can be cut out from a sheet of 1000 cm x 500 cm.

Sheet size = 1000 cm x 500 cm

Size of the rectangular piece to be cut = 50 cm x 20 cm

No. of pieces to be cut in lengthwise = $\frac{1000}{50} = 20$

No. of pieces to be cut in breadthwise = $\frac{500}{20} = 25$

Total no. of pieces to be cut out = 20 x 25

= 500

5 The perimeter of a rectangle is 320 metre. Its sides are in the ratio of 5:3. Find the area of the rectangle.

Ratio = 5:3 = l : b

length l = $5x$

breadth b = $3x$

2(l + b) = Perimeter

$2(5x + 3x) = 320$

$2(8x) = 320$

$16x = 320$

$x = \frac{320}{16} = 20$

l = $5x$ = 5 x 20 = 100 m

b = $3x$ = 3 x 20 = 60 m

Area = l x b (length = 100m, breath = 60m)

= 100 x 60

Area = 6000 m^2

Assignment

1 Find the area of a rectangular plot whose sides are 24 metres and 20 metres respectively. Also find the perimeter of the plot.

2 How many rectangular pieces of 5 cm x 4 cm will you get out of 65 cm x 30 cm brass sheet?

3 Find its breadth and area if the perimeter of a rectangle is 400 metre and its length is 140 m. .

4 Find its area, if the opposite sides of a rectangle are 64 cm and 25 cm respectively.

5 What is the width of the rectangle if a rectangle has an area of 224 cm^2 and length 16 cm.

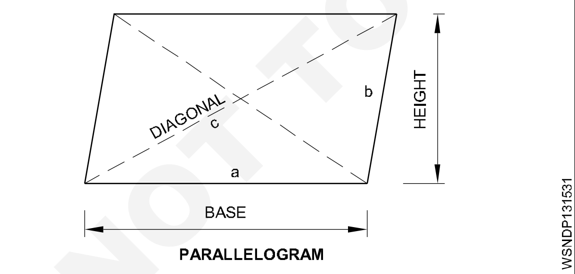
6 What is the length of the diagonal of a rectangle with sides 16 cm and 12 cm?

7 Find the area of the rectangle if the perimeter of the rectangle is 100 cm and the ratio of its length and breadth is 3:2.

Parallelogram

This is also a four sided figure, opposite side being parallel to each other.

Fig 1



Area of parallelogram = base x height

$$\text{or} = 2x\sqrt{s(s-a)(s-b)(s-c)}$$

Where

$$s = \frac{a+b+c}{2}$$

a and b are adjacent sides.

P = 2(a+b)

Examples

1 The base and height of a parallelogram are 7.1 cm and 2.85 cm. Calculate its area.

A = base x height $units^2$

= 7.1 x 2.85

= 20.235 cm^2

2 Find the height of a parallelogram whose area is 20 cm^2 and base is 10 cm.

A = base x height $units^2$

$h = \frac{area}{base}$

$= \frac{20}{10}$

= 2 cm

3. Two sides of a parallelogram are 12 cm and 8 cm. The diagonal is 10 cm long. Find the area of the parallelogram.

Area A $= 2x\sqrt{s(s-a)(s-b)(s-c)}$ units²

s $= \dfrac{a+b+c}{2}$

$= \dfrac{12+8+10}{2}$

$= \dfrac{30}{2}$

$= 15$

A $= 2 \times \sqrt{15(15-12)(15-8)(15-10)}$

$= 2 \times \sqrt{15 \times 3 \times 7 \times 5}$

$= 2 \times \sqrt{1575}$

$= 2 \times 39.686$

Area A $= 79.37\ cm^2$

Assignment

1 Find the area of a parallelogram, if its base and height are 8.1 cm and 30.8 cm respectively.

2 Find the area of a parallelogram, if the sides of a field in the shape of parallelogram are 12 m and 17 m and one of the diagonal is 25 m.

3 Find the base of a parallelogram whose height is 12 cm and area is 120 cm^2.

4 Find the height of a parallelogram whose base is 40 cm and area is 320 cm^2.

5 Find the area of the land if the sides of a land in the shape of a parallelogram are 24 m and 28 m respectively and one of the diagonal is 30 m.

6 What is the perimeter of parallelogram if base is 10 cm and other side is 5 cm?

7 Find the area of parallelogram if its base and height are 25 cm and 12 cm.

8 Find the base of a parallelogram if height is 15 cm and area is 150 cm^2.

9 Find the height of a parallelogram if base is 80 cm and area is 640 cm^2.

10 Find the area of parallelogram if its base and height are 15 cm and 8 cm.

11 Calculate the perimeter and area of parallelogram if base, height are 12.7 cm, 5.5 cm and other side is 6.5 cm.

12 Find the height of parallelogram if the area is 20 cm^2 and base is 10 cm.

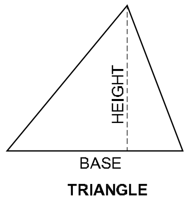
Mensuration - Area and perimeter of triangles

Triangles

Tri means three. Hence tri- angle means three angled figure. For construction of three angled figure, there should be three sides. Hence triangle means three sided figure. Sum of the three angles of any triangle = 180°.

i Any triangle.

Fig 1



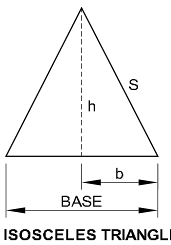
WSNDP131611

Area of any triangle = $\frac{1}{2}$ x Base x Height unit^2

ii Isosceles Triangle

In this triangle two of its sides are equal.

Fig 2



WSNDP131612

Area of isosceles triangle = $\frac{1}{2}$ x Base x Height

Where

base = 2.b

s = One of equal sides (or) Slant height

h = Height = $\sqrt{s^2 - b^2}$

Area of isosceles triangle = $\frac{1}{2}$ x 2b x $\sqrt{s^2 - b^2}$

= b . $\sqrt{s^2 - b^2}$ unit^2

(Where b= half of base)

(or) Area of Isosceles triangle = $\frac{1}{4} b \sqrt{4a^2 - b^2}$ unit^2

a = Equal sides

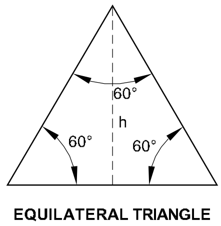
b = Base

iii Equilateral triangle

In this triangle all the three sides are equal. Hence angle between adjacent sides is 60° (Three angles total = 180°);

angle between sides = $\frac{180}{3}$ = 60°

Fig 3



WSNDP131613

Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ x side^2

= $\frac{\sqrt{3}}{4}$ x a^2 unit^2

Where $\sqrt{3}$ = 1.732

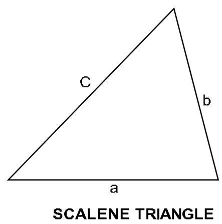
Perimeter P = 3a unit

P = $\frac{\sqrt{3}}{2}$a unit

iv Scalene triangle

In this triangle the sides are not equal. Angles between the sides, are also not equal. we may also call this triangle as irregular triangle.

Fig 4



WSNDP131614

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ unit^2

where

a,b,c are sides of triangle

s = Semi perimeter = $\frac{a+b+c}{2}$ unit

v Right angled triangle

In this triangle, angle between one of two adjacent sides is 90^0. Right angle means 90°. That's why right angled triangle means, one of the angles of this triangle is definitely 90°.

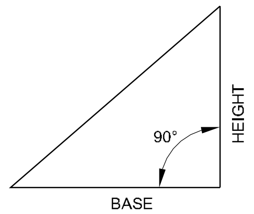
Area of right angled triangle

= $\frac{1}{2}$ x Base x Height

= $\frac{1}{2}$ bh unit^2

Hypotenuse = $\sqrt{Base^2 + Height^2}$

Fig 5



RIGHT ANGLED TRIANGLE

WSNDP131615

Where hypotenuse means, the diagonal or largest length of the side of right angled triangle.

Examples

1 Calculate its area if the base and height of a Right angled triangle are 10 cm and 3.5 cm respectively.

Base (b) = 10 cm

Height (h) = 3.5 cm

Area (A) = ?

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 3.5$$

$$\mathbf{= 17.5\ cm^2}$$

2 Calculate the base of a triangle having an area of 15 cm^2 and height is 3.5 cm.

Area (A) = 15 cm^2

Height (h) = 3.5 cm

Base (b) = ?

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times b \times 3.5 = 15$$

$$\mathbf{b} = \frac{15 \times 2}{3.5}$$

$$\mathbf{= 8.57\ cm}$$

3 Calculate the height of a triangle whose area is 60 cm^2 and base is 10 cm.

Area (A) = 60 cm^2

Base (b) = 10 cm

Height (h) = ?

$$\frac{1}{2} \times b \times h = A$$

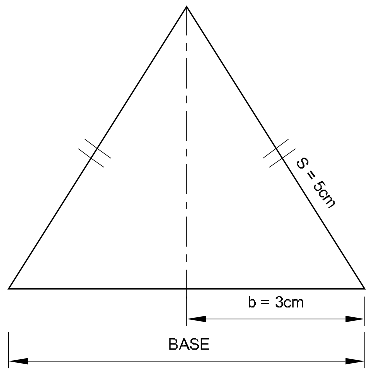
$$\frac{1}{2} \times 10 \times h = 60$$

$$h = \frac{60 \times 2}{10}$$

height h = 12 cm

4 Find the area of an isosceles triangle whose base is 6 cm long and each of the other two sides 5 cm long.

Fig 6



ISOSCELES TRIANGLE

WSNDP131616

Base (b) = 6 cm = $\frac{6}{2}$ = 3 cm

Equal sides or slant height 's' = 5 cm

Area (A) = ?

$$A = b \times \sqrt{s^2 - b^2}$$

$$= 3 \times \sqrt{5^2 - 3^2}$$

$$= 3 \times \sqrt{25 - 9}$$

$$= 3 \times \sqrt{16}$$

$$= 3 \times 4$$

$$= 12\ cm^2$$

or

$$A = \frac{1}{4} b \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 6 \sqrt{4 \times 5^2 - 6^2}$$

$$= \frac{1}{4} \times 6 \times 8$$

$$\mathbf{= 12\ cm^2}$$

5 Find its height if an isosceles triangle has base of 200 mm and its area is 2000 mm^2.

Base = 200 mm

Area = 2000 mm^2

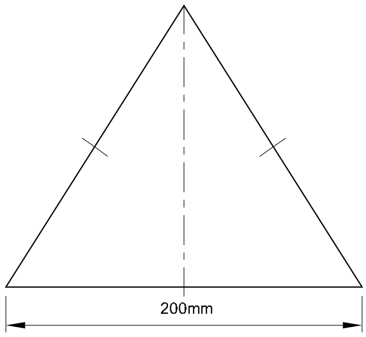
h = ?

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times 200 \times h = 2000$$

$$h = \frac{2000 \times 2}{200} \mathbf{= 20\ mm}$$

Fig 7



ISOSCELES TRIANGLE

6 Find the area of an equilateral triangle whose side is 5 cm.

Area $= \frac{\sqrt{3}}{4} a^2$ unit^2

$= \frac{1.732}{4} \times 5 \times 5$

= 10.825 cm²

7 Calculate its perimeter if one side of an equilateral triangle is 55 mm long.

Side = 55 mm

Perimeter (P) = ?

P = 3a unit

= 3 x 55

= 165 mm

8 Find the area of the triangle having its sides are 9cm, 10cm and 12 cm.

Semi Perimeter $= \frac{a+b+c}{2}$ unit

$= \frac{9+10+12}{2} = \frac{31}{2}$

= 15.5 cm

Area A $= \sqrt{s(s-a)(s-b)(s-c)}$ unit^2

$= \sqrt{15.5(15.5 - 9)(15.5 - 10)(15.5 - 12)}$

$= \sqrt{15.5 \times 6.5 \times 5.5 \times 3.5}$

$= \sqrt{1939.4375}$

= 44.03 cm²

9 Find the cost of polishing on both sides of a triangular metal plate has sides 60 cm, 50 cm and 20 cm at the rate of Rs.1.35 per 100 cm²

Semi Perimeter $= \frac{a+b+c}{2}$ unit

$= \frac{60+50+20}{2} = \frac{130}{2}$

= 65 cm

Area A $= \sqrt{s(s-a)(s-b)(s-c)}$ unit^2

$= \sqrt{65(65 - 60)(65 - 50)(65 - 20)}$

$= \sqrt{65 \times 5 \times 15 \times 45}$

= 468.4 cm²

Area of polish on both sides = 2 x 468.4

= 936.8 cm²

Cost of polish per 100 cm² = Rs. 1.35

∴ Cost of polish is 936.8 cm² $= \frac{936.8}{100} \times 1.35$

= Rs. 12.65

10 Find the area of the right angled triangle with base 20 cm and height 8 cm.

Base b = 20 cm

Equal sides or slant height = 8 cm

Area (A) = ?

Area (A) $= \frac{1}{2} \times$ base x height unit^2

$= \frac{1}{2} \times 20 \times 8$

= 80 cm²

11 Find the area of the right angled triangle if the sides containing the right angle being 10.5 cm and 8.2 cm.

Area (A) $= \frac{1}{2} \times$ base x height unit^2

$= \frac{1}{2} \times 10.5 \times 8.2$

= 43.05 cm²

12 Calculate the perpendicular height of the triangle if the area of the right angled triangle is 19.44 m² and its one of the adjacent side containing the right angle being 5.4 m.

$\frac{1}{2} \times$ base x height unit^2 = Area

$\frac{1}{2} \times 5.4 \times h$ = 19.44

$$h = \frac{19.44 \times 2}{5.4}$$

$$\mathbf{= 7.2\ m}$$

13 Calculate the base of a right angled triangle having an area of 12.5 cm^2. If its height is 2.5 cm.

$$\frac{1}{2} \text{ x base x height unit}^2 = \text{Area}$$

$$\frac{1}{2} \text{ x b x } 2.5 = 12.5$$

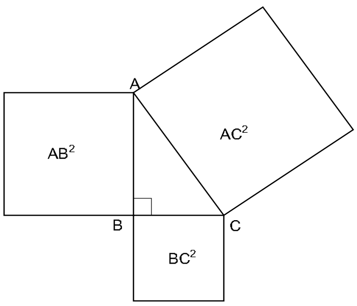
$$b = \frac{12.5 \times 2}{2.5}$$

$$\mathbf{= 10\ cm}$$

Pythagoras theorem

In a right angled triangle the area of the square drawn with the hypotenuse as the side is equal to the sum of the areas of the squares drawn with the other two sides.

Fig 8



PYTHAGORAS THEOREM

WSNDP131618

$\angle B = 90^\circ$

AC = Hypotenuse

AB & BC = Adjacent sides

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\therefore \quad AC = \sqrt{AB^2 + BC^2}$$

1 Calculate the hypotenuse of a right angled triangle whose base is 5 cm and height is 12 cm.

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 12^2 + 5^2$$

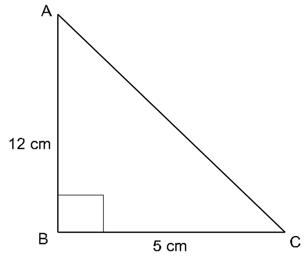
$$= 144 + 25$$

$$= 169$$

$$AC = \sqrt{169}$$

$$\mathbf{= 13\ cm}$$

Fig 9

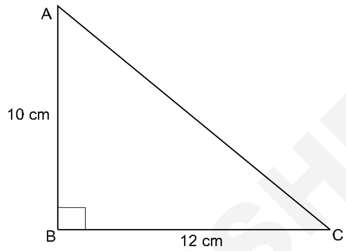


PYTHAGORAS THEOREM

WSNDP131619

2 What is the length of the hypotenuse of a right angled triangle, when the sides containing the right angles are 10 cm and 12 cm.

Fig 10



PYTHAGORAS THEOREM

WSNDP13161A

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 10^2 + 12^2$$

$$= 100 + 144$$

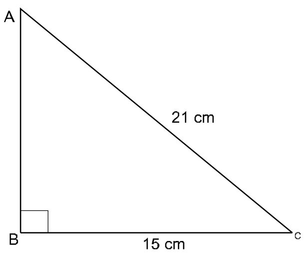
$$= 244$$

$$AC = \sqrt{244}$$

$$\mathbf{= 15.62\ cm}$$

3 Find the height of a right angled triangle whose base is 15 cm and hypotenuse is 21 cm.

Fig 11



PYTHAGORAS THEOREM

WSNDP13161B

As per pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 15^2 = 21^2$$

$$AB^2 = 441 - 225$$

$$= 216$$

$$AB = \sqrt{216}$$

$$\mathbf{= 14.7\ cm}$$

Mensuration - Area and perimeter of circle, semi-circle, circular ring, sector of circle, hexagon and ellipse

Circle

It is the path of a point which is always equal from its centre is called a circle.

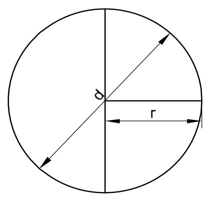
r = radius of the circle

d = diameter of the circle

$\pi = \frac{22}{7} = 3.14$

Area of the circle = πr^2

Fig 1



PROPERTIES OF CIRCLE

(or) $= \frac{\pi}{4} d^2 \text{ unit}^2$

Circumference of the circle $2\pi r$ (or) πd unit

Examples

1 Find the area of a circle whose radius is 1.54 m. Also find its circumference.

radius r = 1.54 cm

Area A = ?

Circumference C = ?

A $= \pi r^2 \text{ unit}^2$

$= \frac{22}{7} \times 1.54 \times 1.54$

$= 7.4536 \text{ m}^2$

C $= 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 1.54$

= 9.68 m

2 Find out the circumference if the area of a circular shape of land is 616 m^2.

A $= \pi r^2 \text{ unit}^2$

$r^2 = \frac{616}{\pi}$

$= \frac{616 \times 7}{22}$

= 196

r $= \sqrt{196}$

= 14 m

Circumference $= 2\pi r$ unit

$= 2 \times \frac{22}{7} \times 14$

= 88 m

3 Find the side of square into which it can be bent if a wire is in the form of a circle of radius 49 cm.

radius of circle r = 49 cm

side of square = ?

Perimeter of the square = Perimeter of the circle

4a $= 2\pi r$

4a $= 2 \times \frac{22}{7} \times 49$

4a = 308

a $= \frac{308}{4}$

= 77 cm

4 Find its radius if the difference between the circumference and diameter of a circle is 28 cm.

Circumference - Diameter = 28 cm

$2\pi r - d = 28$

$2\pi r - 2r = 28$

$2r(\pi - 1) = 28$

$2r\left(\frac{22}{7} - 1\right) = 28$

$2r\left(\frac{22-7}{7}\right) = 28$

$2r \times \frac{15}{7} = 28$

$r = \frac{28 \times 7}{15 \times 2}$

= 6.53 cm

5 What is the side of the largest square cut out from a circle of 50 cm dia.?

Diagonal of a square = Diameter of the circle

$\sqrt{2}a = 50$

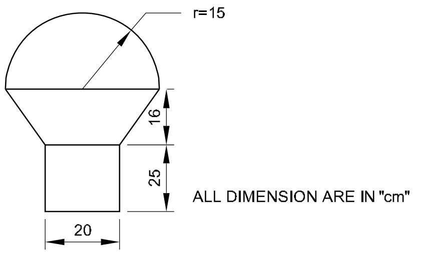
$a = \frac{50}{\sqrt{2}}$

$= \frac{50}{1.414}$

= 35.36 cm

6 Calculate the area of the figure given below.

Fig 2



PROPERTIES OF CIRCLE

WSNDP131712

Area of rectangle = lb unit^2

$= 25 \times 20$ cm^2

$= 500$ cm^2

Area of Trapezium = $\frac{1}{2} \times (a + b)\,h$

$= \frac{1}{2} \times (30 + 20)\,16$ cm^2

$= \frac{1}{2} \times 50 \times 16$ cm^2

$= 400$ cm^2

Area of Semi circle = $\frac{\pi r^2}{2}$ unit^2

$= \pi \times 15^2 \times \frac{1}{2}$ cm^2

$= 353.57$ cm^2

Total area of the figure = 500 + 400 + 353.57

= 1253.57 cm²

7 Find the area of remaining steel plate if in a rectangular steel plate 16 cm x 12 cm, there are 6 holes each 4 cm in diameter.

Area of a rectangular plate = length x breadth unit^2

$= 16 \times 12$

$= 192$ cm^2

No. of holes = 6

Radius of hole = 2 cm

Area of 6 holes = $6 \times \pi r^2$ unit^2

$= 6 \times \frac{22}{7} \times 2 \times 2$ unit^2

$= 75.43$ cm^2

Area of remaining plate = 192 - 75.43

= 116.57 cm²

— — — — — — — —

Semi circle

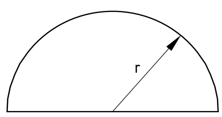
A semi circle is a sector whose central angle is 180°. Length of arc of semi circle.

Length of arc $\ell = 2\pi r \times \frac{180}{360}$

$= 2\pi r \times \frac{1}{2} = \pi r$ unit

Area of semi circle = $\frac{\pi r^2}{2}$ Sq. units

Fig 3



SEMI CIRCLE

WSNDP131713

Perimeter of a semi circle = $\frac{2\pi r}{2} + 2r$

$= \pi r + 2r$

$= r(\pi + 2)$ unit

Examples

1 Calculate the circumference and area of a semi circle whose radius is 6 cm.

radius r = 6 cm

Area A = ?

Circumference c = ?

$A = \frac{\pi r^2}{2}$ unit^2

$= \frac{22}{7} \times \frac{1}{2} \times 6^2$

Area (A) $= \frac{22}{7} \times \frac{1}{2} \times 36$

$= \frac{396}{7} = 56.57$ cm^2

Perimeter of a semicircle = $\frac{2\pi r}{2} + 2r = \pi r + 2r$

$= r(\pi + 2) = 6\left(\frac{22}{7} + 2\right)$

$= 6\left(\frac{22+14}{7}\right)$

$= 6 \times \frac{36}{7}$

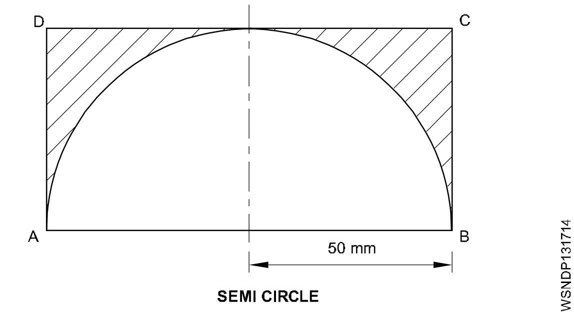
$= \frac{216}{7}$

= 30.86 cm

2 From the figure given below ABCD is a steel plate, a semi circular plate of radius 50 mm has been prepared by gas cutting. Find the waste area.

Plate length AB	= 100 mm
Breadth BC	= 50 mm
Radius	= 50 mm

Fig 4



Waste area = Plate area - Area of semi circle

$= lb - \frac{\pi r^2}{2}$

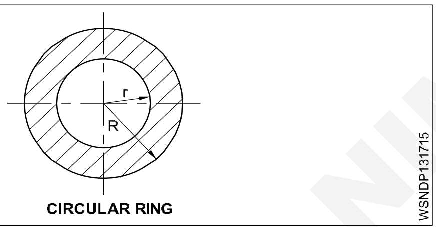
$= 100 \times 50 - \frac{22 \times 50 \times 50}{7 \times 2}$

= 5000 - 3928.57

= 1071.43 mm²

Circular ring

Fig 5



R = Outer radius of circular ring

r = Inner radius of circular ring

Area of circular ring = $\pi (R^2 - r^2)$ unit²

or

A = $\pi (R + r)(R - r)$ unit²

1 Calculate the area of cross section of pipe having outside dia of 17 cm and inside dia of 14 cm.

Given:

Outer dia of pipe = 17 cm

Outer radius of pipe (R) = $\frac{17}{2}$ = 8.5 cm

Inner dia of pipe = 14 cm

Inner radius of pipe (r) = $\frac{14}{2}$ = 7 cm

To find:

Area of cross section of pipe = ?

Solution:

Area of cross section of pipe $= \pi (R + r)(R - r)$ unit²

$= \pi (8.5 + 7)(8.5 - 7)$

$= \frac{22}{7} \times 15.5 \times 1.5$ cm²

= 73 cm²

2 Find the distance between the boundaries and the area of the circular ring, if the circumference of two concentric circle are 134 cm and 90 cm.

Given:

Circumference of outer circle = 134 cm

Circumference of inner circle = 90 cm

To find:

Distance between the circles = ?

Area of circular ring = ?

Solution:

Circumference of outer circle = 134 cm

$2\pi R$ = 134 cm

$R = \frac{134}{2\pi} = 21.32$cm

Circumference of inner circle = 90 cm

$2\pi r$ = 90 cm

$r = \frac{90}{2\pi} = 14.32$ cm

Distance between the circle = R - r

= 21.32 - 14.32 cm

= 7 cm

Area of circular ring $= \pi (R + r)(R - r)$ unit²

$= \pi (21.32 + 14.32)(21.32 - 14.32)$ cm²

$= \frac{22}{7} \times 35.64 \times 7$ cm²

= 784.08 cm²

3 A wire can be bend in the form of a circle of radius 56 cm. If it is bend in a form of a square, find the side.

Given:

Radius of circle = 56 cm

To find:

Side of square = ?

Solution:

Radius of circle = 56 cm

Circumference of circle $= 2\pi r$ unit $= 2\pi \times 56$ cm

Side of square $= x$ cm

Wire can be bend from the form of round to square

Perimeter of square = circumference of circle

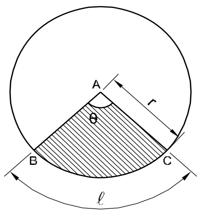
$4 \times a = 352$ cm

$a = \frac{352}{4} = 88$ cm

— — — — — — —

Sector of Circle

Fig 6



SECTOR OF CIRCLE

WSNDP131716

θ = Angle of sector of circle

l = Arc length

r = radius

Length of Arc $\ell = \frac{\theta}{360^0} \times 2\pi r$ unit

Perimeter $P = 2r + \ell$ unit

Area $= \frac{\theta}{360^0} \times \pi r^2$ unit² (or) $A = \frac{\ell r}{2}$ unit²

1 Find the perimeter and area of a sector of circle of radius 7 cm and its angle is 120⁰.

Given:

Angle of sector of circle $= 120^0$

Radius = 7 cm

To find:

Perimeter = ? , Area = ?

Solution:

Length of arc $(\ell) = \frac{\theta}{360^0} \times 2\pi r$ unit

$= \frac{120}{360} \times 2 \times \frac{22}{7} \times 7$ cm

= 14.67 cm

Perimeter $= 2r + \ell$ unit

$= 2 \times 7 + 14.67$ cm

= 28.67 cm

Area $= \frac{\theta}{360^0} \times \pi r^2$ unit²

Area $= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 7^2$ cm² = 51.33 cm²

2 Find the radius of the circle if the angle is 60⁰ and the area of a sector of a circle is 144 cm²,

Given:

Area of sector of circle (A) = 144 cm²

Angle of sector of circle $\theta = 60^0$

To find:

Radius of circle = ?

Solution:

Area (A) $= \frac{\theta}{360^0} \times \pi r^2$ unit²

$144 = \frac{60^0}{360^0} \times \frac{22}{7} \times r^2$ cm²

$r^2 = 274.91$ cm²

$r = \sqrt{274.91} = 16.58$ cm

3 Find the area of the sector whose angle is 105⁰, and the perimeter of sector of circle is 18.6 cm.

Given:

Perimeter of a sector of a circle = 18.6 cm

Angle of sector of circle = 105⁰

To find:

Area = ?

Solution:

Length of Arc $(\ell) = \frac{\theta}{360^0} \times 2\pi r$ unit

$\ell = \frac{105^0}{360^0} \times 2 \times \frac{22}{7} \times r$

$= 1.83r$

Perimeter (P) $= \ell + 2r$ unit

$18.6 = 1.83r + 2r$

$3.83r = 18.6$ cm

$r = \frac{18.6}{3.83} = 4.86$ cm

$$\text{Area A} = \frac{\theta}{360^0} \times \pi r^2 \text{ unit}^2$$

$$= \frac{105^0}{360^0} \times \frac{22}{7} \times (4.86) \text{ cm}^2$$

$$= 21.65 \text{ cm}^2$$

4 Find the area, if the radius is 12.4 cm and the perimeter of a sector of a circle is 64.8 cm.

Given:

Perimeter P = 64.8 cm

Radius r = 12.4 cm

To find:

Area A = ?

Solution:

Perimeter P = ℓ + 2r unit

ℓ = P - 2r unit

= 64.8 - 2 (12.4) cm

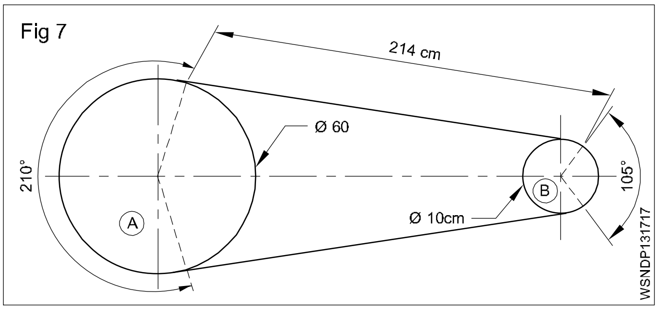
= 64.8 - 24.8 = 40 cm

$$\text{Area A} = \frac{\ell r}{2} \text{ unit}^2 = \frac{40 \times 12.4}{2}$$

$$= 248 \text{ cm}^2$$

5 Find out the length of the belt , if the arrangement of a belt is shown in the figure below.

Fig 7



Solution:

$$\text{Length } \ell_A = \frac{\theta}{360^0} \times 2\pi r \text{ unit}$$

$$= \frac{210^0}{360^0} \times 2 \times \frac{22}{7} \times 30 = 110 \text{ cm}$$

$$\text{Length } \ell_B = \frac{\theta}{360^0} \times 2\pi r \text{ unit}$$

$$= \frac{105^0}{360^0} \times 2 \times \frac{22}{7} \times 5 = 91.7 \text{ cm}$$

$= \ell_A + \ell_B + 2 \times 214$ cm

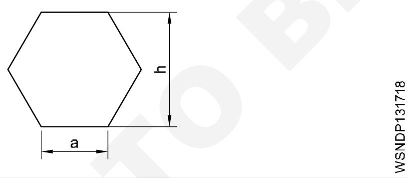
= 110 + 9.17 + 428 cm

= 547.17 cm

— — — — — — —

Hexagon

Fig 8



Side = a unit

Perimeter P = 6a unit

Area A = $6 \times \frac{\sqrt{3}}{4} \times a^2$ units² (Area of 6 equilateral triangle)

DAF (Distance Across Flats) = $\sqrt{3} \times a$ unit

DAC (Distance Across Corners) = 2 x a unit

1 Find out the perimeter, area, DAF and DAC of a regular hexagon whose side is 2cm.

(DAF - Distance Across Flats)

(DAC - Distance Across Corners)

Given: Side of hexagon (a) = 2cm

To Find: P = ?, A = ?, DAF = ?, DAC = ?

Solution:

Perimeter of hexagon (P) = 6a unit

= 6a unit = 6 x 2 cm = 12 cm

Area of hexagon A $= 6 \times \frac{\sqrt{3}}{4} \times a^2$ unit²

$$= 6 \times \frac{1.732}{4} \times 2^2$$

= 10.392 cm²

DAF (Distance Across Flats) = $\sqrt{3} \times a$ unit

= $\sqrt{3} \times 2$ = 1.732 x 2

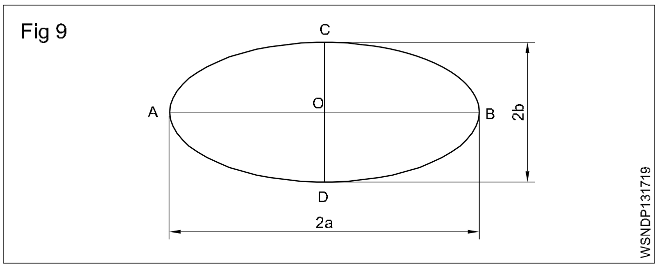
= 3.464 cm

DAC (Distance Across Corners) = 2 x a unit

= 2 x 2 = 4 cm

— — — — — — —

Ellipse



Major axis AB = 2a

Half of Major axis OB = a,

Minor axis CD = 2b

Half of Minor axis OC = b

Area of ellipse A = π x a x b unit^2

Perimeter of ellipse P = $2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

1 Find its area and perimeter, if the major and minor axis of an ellipse are 12 cm and 8 cm respectively.

Solution:

Major axis 2a = 12 cm

a $= \frac{12}{2} = 6$ cm

Minor axis 2b = 8 cm

b $= \frac{8}{2} = 4$ cm

Area A = π x a x b unit^2

$= \frac{22}{7}$ x 6 x 4 cm^2

= 75.43 cm^2

Perimeter (P) $= 2\pi\sqrt{\frac{(a^2+b^2)}{2}}$ unit

$= 2\times\frac{22}{7}\sqrt{\frac{(6^2+4^2)}{2}}$ unit

$= 2\times\frac{22}{7}\sqrt{\frac{36+16}{2}}$ unit

$= 2\times\frac{22}{7}\times\sqrt{26}$

= 2 x $\frac{22}{7}$ x 5.1 = 32.06 cm

— — — — — — —

Assignment

Circle

1 Find the circumference and area of a circle whose radius is 10 metre.

2 Find its diameter if the area of a circle is 330 cm^2.

3 Find its area if the circumference of a circle is 50 cm.

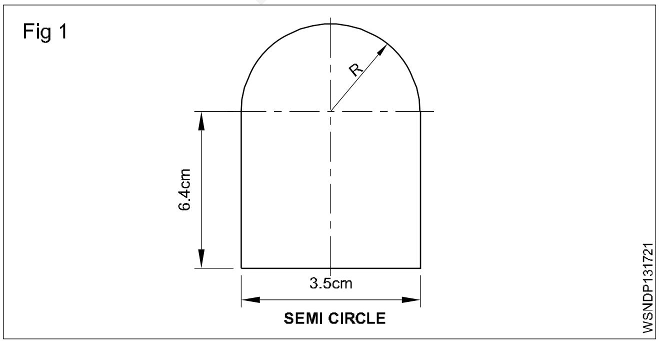
4 Find out the area and circumference of a circle of diameter is 50 cm.

5 Find its area if the circumference of a circle is 44 cm.

Semi circle

1 Calculate the circumference and area of semi circle whose radius is 14 cm.

2 Find area of the given figure.



Circular ring

1 Find out area of a ring washer, whose inner radius and outer radius are 13 cm and 15 cm respectively.

2 Find the area of a ring portion of a washer whose outer dis is 30 m and inner dis is 20 m. Also calculate the difference between the circumference of circles.

Sector of circle

1 Find the perimeter and area of a sector of a circle of radius 5cm and its angle is 96^0.

2 Find the radius of the circle if the angle is 90^0 and the area of sector of a circle is 196 cm^2.

Hexagon

1 Find out the Area, perimeter, DAF, and DAC of hexagon of side 4cm.

2 Find the area of cross section of a regular hexagon rod whose side is 7.5 cm.

Ellipse

1 Find the area of the biggest ellipse that can be inscribed in a rectangle of length 18 cm and breadth 12 cm. Also calculate its perimeter.

2 How much fencing will be required to enclose an elliptical plot of ground the axes of the ellipse being 200 and 170 meter respectively.

Mensuration - Surface area and volume of solids - cube, cuboid, cylinder, sphere and hollow cylinder

Cube

All sides of cube are same i.e length,breadth and height have same value. It is bounded by six equal square faces.

Volume of cube = side x side x side

= a^3 unit^3

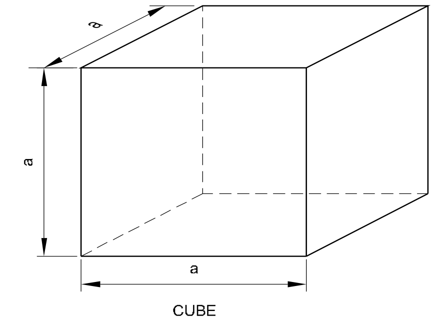
Lateral surface area = $4a^2$ unit^2

Total surface area = 6 x side x side

= $6a^2$ unit^2 $\sqrt{3} = 1.732$

Diagonal d = $\sqrt{3}a$

Fig 1



Rectangular solid (or) cuboid

Rectangular soild is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.

Volume of rectangular solid

= Length x breadth x height

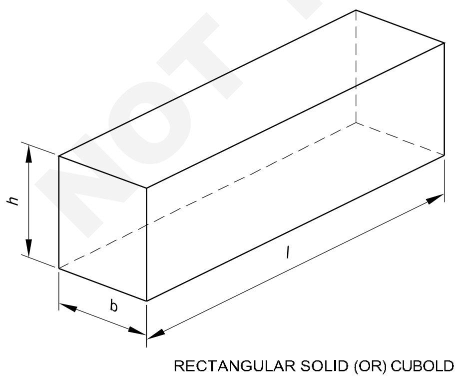
= l . b . h unit^3

Lateral surface area = 2h(l+b) unit^2

Total surface area = 2lb + 2bh + 2hl

= 2(lb+bh+hl) unit^2

Fig 2



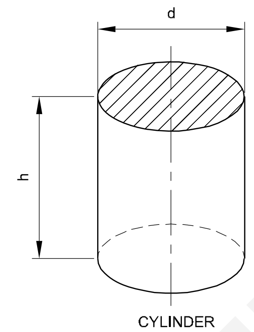
l = length, b = breadth and h = height

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$

Fig 3



Curved area of cylinder = $2\pi rh$ unit^2

Total surface area of cylinder = $2\pi r(h+r)$ unit^2

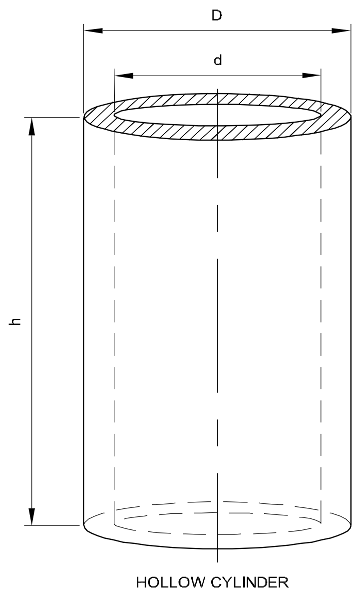
r = Radius of base , d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.

Fig 4



Volume of hollow cylinder = $\pi (R^2 - r^2)$ h (or)

= $\pi (R + r) (R - r)$ h (or)

= $\frac{\pi}{4} (D^2 - d^2)$ h unit^3

= $\frac{\pi}{4}(D + d)(D - d)h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

Mean dia = $\frac{D - d}{2}$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Finding out volumes of solids

The space occupied by a body is known its volume. The volume of a body indicates the capacity to hold substance in it.

The general form of Lateral surface area Total surface area and Volume is :

Lateral surface area = perimeter of the base x height

Total surface area = LSA + 2 (base area)

Volume = Area of base x height

Important and commonly used solids are described below one after another:

Cube

1 Find the diagonal, lateral surface area,, total surface area and volume of a cube of side 4.5 cm.

side a = 4.5 cm

diagonal d = $\sqrt{3}$a unit

= 1.732 x 4.5

= 7.794 cm

L.S.A = $4a^2$ unit^2

= 4 x 4.5 x 4.5

= 81 cm^2

T.S.A = $6a^2$ unit

= 6 x 4.5 x 4.5

= 121.5 cm^2

V = a^3 unit^3

= 4.5 x 4.5 x 4.5

= 91.125 cc.

2 Calculate volume of a cube where side is 9 cm

a = 9 cm

V = ?

V = a^3

= 9 x 9 x 9

= 729 cm^3

3 Find out side of the cube if a cube has volume of 3375cm^3.

V = 3375 cm^3

a = ?

a^3 = 3375

a = $\sqrt[3]{3375}$

= $\sqrt{3x3x3x5x5x5}$

= 3 x 5

= 15 cm

4 Find the side of a cube, if its surface area is 216 cm^2

Surface area = T.S.A = 216 cm^2

$6a^2$ = 216

a^2 = $\frac{216}{6}$

= 36

a = $\sqrt{36}$

= 6 cm

5 Find the side of the square tank, if its height is 2 metre and has the capacity to hold 50,000 litre of water.

Height of square shape tank (h) = 2 m

Capacity = 50,000 litre

1000 litre = 1 m^3

50,000 Litre = $\frac{50000}{1000}$

= 50 m^3

Capacity of tank = 50 m^3

a^2 x h = 50

a^2 x 2 = 50

a^2 = $\frac{50}{2}$ = 25 m^2

a = $\sqrt{25}$ = 5 m

Side of the square tank = 5 m

Rectangular Solid (or) Cuboid

1 Find its volume and T.S.A if a tank is 20 m long, 15 m broad and 12 m high.

l = 20m

b = 15 m

h = 12 m

v = ?

T.S.A = ?

Volume v = lbh unit^3

= 20 x 15 x 12

= 3600 m^3

T.S.A = 2(lb + bh + hl) unit^2

= 2((20 x 15) + (15 x 12) + (20 x 12))

= 2 (300 + 180 + 240)

= 1440 m^2

2 Find out its height if the cross section is 260 mm length and 180 mm wide rectangular and the capacity of a fuel tank is 10500 cm^3.

l = 260mm = 26 cm

b = 180 mm =18 cm

v = 10500 cm^3

h = ?

l.b.h = volume

26 x 18 x h = 10500

$h = \frac{10500}{26 \times 18}$

= 22.44 cm

3 How many litres of water it can store if a water tank has the following dimensions length = 1 metre, width = 0.8 metre and height = 1.2 metre?

Volume = l x b x h unit^3

= 1 x 0.8 x 1.2

= 0.96 m^3 [1 m^3 = 1000 litres]

= 0.96 x 1000

= 960 litres of water can store in the tank.

4 Find its volume if the base of a prism is a rectangle having 5m length, 4m breadth and the height of the prism is 15m.

The base of prism is rectangle

Area of base = length x breadth

= 5 x 4

= 20 square m

Volume of prism = Area of base x Height

= 20 x 15

= 300 cm^3

Cylinder

1 Find the volume and total surface are of a cylinder having 9cm diameter and 15 cm height.

diameter = 9 cm

radius r = 4.5 cm

height h = 15 cm

Volume V = ?

T.S.A = ?

V = $\pi r^2 h$ unit^3

$= \frac{22}{7} \times 4.5 \times 4.5 \times 15$

= 954.4 cm^3

T.S.A = $2\pi r(h+r)$ unit^2

$= 2 \times \frac{22}{7} \times 4.5 (15 + 4.5)$

$= 2 \times \frac{22}{7} \times 4.5 \times 19.5$

= 551.4 cm^2

2 Calculate the radius if the curved surface area of a cylindrical roller is 48π cm^2 and the roller is 10 cm long

C.S.A = 48π cm^2

length = 10 cm

radius = ?

$2\pi rh = 48\pi$

$2 \times \pi \times r \times 10 = 48\pi$

$r = \frac{48 \times \pi}{2 \times \pi \times 10}$

= 2.4 cm

3 Find its radius if the volume of a cylinder is 5544 cm^3 and its height is 16 cm.

$\pi r^2 h = v$

$3.14 \times r^2 \times 16 = 5544$

$r^2 = \frac{5544}{3.14 \times 16}$

$r^2 = \frac{5544}{50.24}$

= 110.35

$r = \sqrt{110.35}$

= 10.5 cm

4 Find the diameter of the tank if the volume of a circular tank is 68.46 m^3, its height is 2 m.

$\pi r^2 h = 68.46$

$r^2 = \frac{68.46}{3.14 \times 2}$

$r^2 = 10.9$

$r = \sqrt{10.9}$

= 3.3 m

diameter = 2r

= 2 x 3.3

= 6.6 m

5 A cylindrical vessel is to be made of 3 metre long and 1.9994 metre diameter. Calculate its total surface area, if it is in a closed form on one end.

h = 3m

d = 1.9994 m

r = 0.9997 m

T.S.A = C.S.A + Base area

$= 2\pi rh + \pi r^2$

$= (2 \times \frac{22}{7} \times 0.9997 \times 3) + (\frac{22}{7} \times 0.99997^2)$

= 18.85 + 3.14

= 21.99 m²

6 How many litres of water a cylinder of radius 75 cm and height 100 cm can hold.

$V = \pi r^2 h \text{ unit}^3$

= 3.142 x 75 x 75 x 100

$= 1767375 \text{ cm}^3$

$= \frac{1767375}{1000}$ [1000 cc = 1 litre]

= 1767.375 litres.

7 Calculate the height of cylindrical tin if a closed rectangular box 40 cm long, 30 cm wide and 25 cm deep has the same volume as that of cylinder tin of radius 17.5 cm.

Volume of cylinder = Volume of rectangular box

$\pi r^2 h = l \times b \times h$

$\frac{22}{7} \times 17.5 \times 17.5 \times h = 40 \times 30 \times 25$

$h = \frac{40 \times 30 \times 25 \times 7}{22 \times 17.5 \times 17.5}$

$= \frac{210000}{6737.5}$

= 31.17 cm

8 An oxygen cylinder is 15 cm in diameter and 100 cm in length. It is filled with gas under pressure so that every cm³ of the cylinder contains 120 cm³ of gas. How much cc of oxygen does this hold?

Volume of cylinder $= \pi r^2 h \text{ unit}^3$

$= \frac{22}{7} \times 7.5 \times 7.5 \times 100$

$= 17678.57 \text{ cm}^3$

Gas contain in 1 cm³ = 120 cm³ of gas

Gas contain in 17678.57 cm³ = 17678.57 x 120

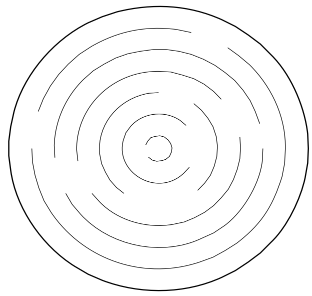
$= 2121428 \text{ cm}^3$

Volume of oxygen = 2121428 cc.

Sphere

Sphere is a solid circular body.

Fig 5



SPHERE

WSNDP131815

Volume of sphere $= \frac{4}{3}\pi r^3$ or

$= \frac{\pi}{6} d^3 \text{ unit}^3$

Total surface area of sphere $= 4\pi r^2 \text{ unit}^2$

Where r = Radius of sphere

d = Diametre of sphere

Radius $= \frac{1}{2}$ of diameter

1 Find the volume and surface area of a sphere of 3 cm radius.

$V = \frac{4}{3}\pi r^3 \text{ unit}^3$

$= \frac{4 \times 22 \times 3 \times 3 \times 3}{3 \times 7}$

$= 113.1 \text{ cm}^3$

Total Surface Area $= 4\pi r^2 \text{ unit}^2$

$= 4 \times \frac{22}{7} \times 3 \times 3$

= 113.1 cm²

2 Find the diameter of sphere having volume of 15625 cc.

$\frac{4}{3}\pi r^3 = \text{Volume}$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 15625$

$r^3 = \frac{15625 \times 3 \times 7}{4 \times 22}$

$$= \frac{328125}{88}$$

$$= 3728.69$$

$$r = \sqrt[3]{3728.69}$$

$$\mathbf{= 15.51\ cm}$$

diameter = 2 x radius

= 2 x 15.51

= 31.02 cm

3 How many spherical balls of 1 cm radius can be made from a sphere of 32 cm diameter.

No. of balls x volume of small sphere = Volume of bigger sphere

$$N \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$N \times \frac{4}{3} \times \pi \times 1^3 = \frac{4}{3} \times \pi \times r^3$$

N = 16 x 16 x 16

= 4096 balls

4 Three brass balls of diameters 3 cm, 4 cm and 5 cm are melted and make into one solid ball, if there is no wastage. Find the diameter of the solid ball.

1st ball d_1 = 3 cm,

r_1 = 1.5 cm

2nd ball d_2 = 4 cm,

r_2 = 2 cm

3rd ball d_3 = 5 cm,

r_1 = 2.5 cm

Diameter of new ball = ?

Volume of new ball = Volume of 3 spherical balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^{\,3} + \frac{4}{3}\pi r_2^{\,3} + \frac{4}{3}\pi r_3^{\,3}$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (1.5^3 + 2^3 + 2.5)^3$$

r^3 = 3.375 + 8 + 15.625

r^3 = 27

$r = \sqrt[3]{27}$

$r = \sqrt[3]{3x3x3}$

r = 3 cm

Diameter of the ball = 2 x r

= 2 x 3

= 6 cm

Assignment

Cube

1 Find the diagonal, lateral surface area, total surface area and volume of cube, whose side is 15 cm.

2 Find the volume of 10 cubes where each side is 5 cm.

3 Find its volume if a solid cube has each of its sides 60 mm long.

4 What is its side if the total surface area of a cube is 384 m^2.

Cuboid

1 Find the volume of the tank in m^3, if the length is 60 m, breadth 40 m and height 20 m.

2 Find the volume of a C.I. casting of a rectangular block having 25 cm x 20 cm x 8 cm size.

3 Calculate the total surface area of a box whose length, width and height are 120 cm, 50 cm and 60 cm respectively.

4 Find the volume of the sheet if a brass sheet is of 25 cm square and 0.4 cm thick.

Cylinder

1 Find the curved surface area of cylinder whose diameter is 18 cm and height 34 cm?

2 Find the total surface area of cylinder whose diameter is 24 cm and height 40 cm?

3 Find out the volume of cylinder whose base is 10 cm radius and height is 40 cm?

Sphere

1 Find the volume of sphere having diameter 3.5cm?

2 Find the total surface area of a sphere having radius 1.75 cm?

3 How many spherical balls of 1 cm radius can be made from a sphere of 16 cm diameter.

4 Three balls of diameter 2m, 4cm and 6 cm are melted and made into one solid ball. If there is no wastage, find the diameter of solid ball.

Mensuration - Finding the lateral surface area, total surface area and capacity in litres of hexagonal, conical and cylindrical shaped vessels

Hexagonal bar

Volume of Hexagonal bar = Area of hexagonal x height

Lateral surface area of hexagonal bar

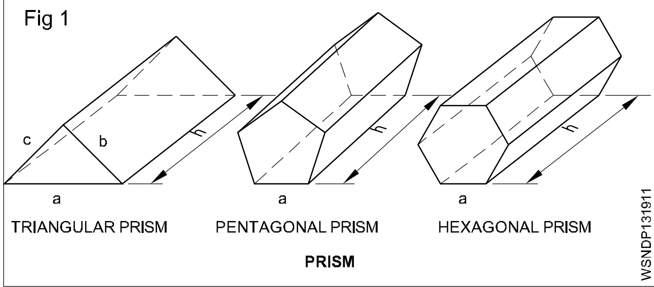
= 6 x side of hexagon x length of the bar

or = 3.464 x length of the bar x flat of hexagon

Total surface area of hexagonal bar

= lateral surface area + (2 x area of hexagon)

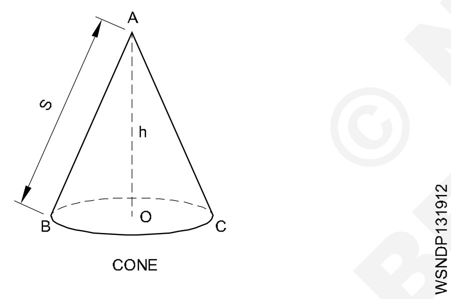
Fig 1



Cone

Cone is a pyramid with a circular base.

Fig 2



Volume of cone = $\frac{1}{3}\pi r^2 h$

or $= \frac{\pi}{12} d^2 h$

Curved area = πrs

Total surface area = $\pi r(s+r)$

Where r = radius of base

d = diametre of base

h = vertical height of cone

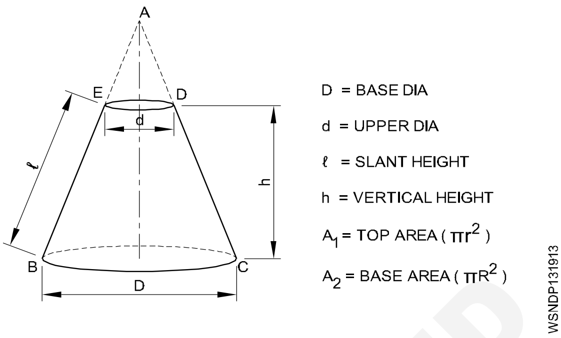
s = slant height $\sqrt{r^2+h^2}$

Frustum of a cone

When a cone is cut by a plane parallel to the base, and upper part is removed, the formation appears, is termed as frustum of a cone. Buckets, oil cans etc.are such frustums in shape.

L.S.A = $\pi l (R + r)$ unit²

Fig 3



TSA = $\pi l (R + r) + A_1 + A_2$ unit²

$V = \frac{\pi}{3} h (R^2 + Rr + r^2)$ unit³

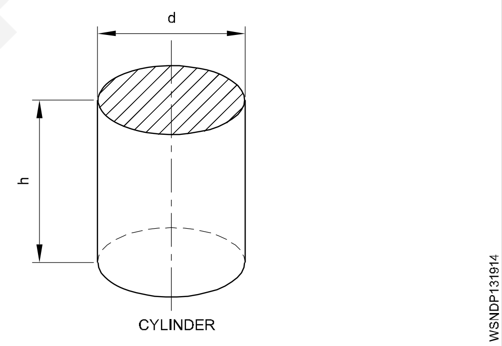
[A_1 = Top area ; A_2 = Bottom area]

Cylinder

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder = $\pi r^2 h$ or $\frac{\pi}{4} d^2 h$

Fig 4



Curved area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r(h+r)$

r = Radius of base, d = Diameter of base

h = Height of cylinder

Hollow cylinder

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.

Volume of hollow cylinder = $\pi (R^2 - r^2) h$ (or)

$= \pi (R + r) (R - r) h$ (or)

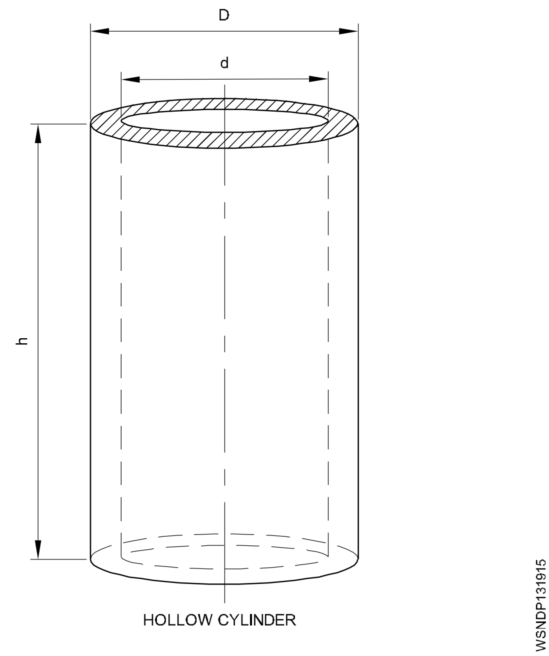
$= \frac{\pi}{4} (D^2 - d^2) h$

$= \frac{\pi}{4} (D + d)(D - d) h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

Fig 5



$\therefore$ TSA : $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

Mean dia = $\frac{D - d}{2}$

If thickness given then:

Volume of hollow cylinder = π x mean dia x thickness x height

Example

1 Find the volume of an hexagonal prism having its side 20 cm and height 200 cm.

Side of hexagonal prism (a) = 20 cm

Height (h) = 200 cm

Volume (V) = Base side area x Height

$= 6 \times \frac{\sqrt{3}}{4} \times a^2 \times h$

$= 6 \times \frac{\sqrt{3}}{4} \times 20 \times 20 \times 200$

$= 1{,}20{,}000 \times \sqrt{3}$

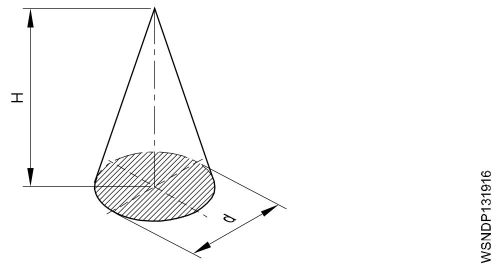
$= 1{,}20{,}000 \times 1.732$

= 2,07,840 cm³

Volume of the hexagonal prism = 2,07,840 cm³

2 Calculate the height. Also find the lateral surface area if a cone has a base diameter of 210 mm and its volume is 3056 cm3.

Fig 6



Volume of a cone = $\frac{1}{3}$ x Area of base x height

$3056\ cm^3 = \frac{1}{3} \times 0.785 \times 210^2 mm^2 \times H$

$H = \frac{3056 \times 3 \times 1000 mm^3}{0.785 \times 210^2\ mm^2} = 264.82$ mm

L = Slant height = $\sqrt{264.83^2 + 105^2} = 284.9mm$

Lateral surface area = $\frac{1}{2}\ \pi \times 210 \times 284.9mm^2$

= 94017 mm² = 940.17 cm²

3 Determine its diameter in mm if the height of a rod of 1.6 metres and its volume is 1.017 metre³.

V = A x H

V = $\pi r^2 \times h$ (or) $\frac{\pi d^2}{4} \times h$

Volume = Area x Height

$= \frac{\pi d^2}{4} = 0.785\ d^2$

$1.017\ m^3 = 0.785 d^2 \times 1.6$ metres

$0.785 d^2 = \frac{1.017}{1.6} m^2$

$d^2 = \frac{1.017}{1.6 \times 0.785} m^2$

$= \frac{1.017}{1.6 \times 785} m^2$

$d = \sqrt{\frac{10170}{16 \times 785}}$ metre

$= \sqrt{\frac{10170}{12560}}$

$= \sqrt{0.8097}$

= 0.8998

= 899.8mm

Trigonometry - Measurement of angles

Introduction:

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

Units:

Measurement of Angles

There are three systems of measuring the angle:

(i) Sexagesimal System

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees.Each part is divided into 60 parts which are called minutes.Each minute is divided into 60 parts which are called seconds.The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

(ii) Centesimal System

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

(iii) Circular System

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

Circumference $= \pi \times$ dia

$= 2\pi r$ (where r is radius of the circle)

$$\pi = \frac{22}{7}$$

Circumference makes an angle $(2\pi r) = 360°$

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie : } \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi \text{ Radian} = 360°$$

$$\pi \text{ Radian} = 180°$$

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1° = \frac{\pi}{180^\circ} \text{ Radian}$$

Examples

1 Convert 45°36'20" into degree and decimal of degree.

60 seconds = 1 minute

$$20 \text{ seconds} = \frac{20}{60} = 0.333'$$

60 minutes = 1 degree

$$36.333 \text{ minutes} = \frac{36.333}{60} = 0.606°$$

$45^0 36'20" = 45.606°$

2 Convert 24.59° into degree, minute and second

1 degree = 60 minutes

0.59 degree = 0.59 x 60 = 35.4'

1 minute = 60 seconds

0.4 minute = 60 sec x 0.4

= 24"

Therefore 24.59° = $24^0 35'24"$

3 Change $50^0 37'30"$ into degrees

By changing angle degrees into decimals

$$30" = \frac{30}{60} = 0.50'$$

37'30" = 37.5'

$$37.5' = \frac{37.5}{60} = 0.625^0$$

$50^0 37'30" = 50.625^0$

4 Convert $23^0 25' 32''$ into radians

We know $1^0 = 60' = 3600''$

Therefore $23^0 25' 32''$

$$= \left(23 + \frac{25}{60} + \frac{32}{3600}\right) \text{ degrees}$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But $180^0 = \pi$ radians

Therefore 23.4255 degrees

$$= \frac{23.4255}{180} \pi \text{ radians}$$

$$= \frac{23.4255}{180} \text{ x } \frac{22}{7} \text{ radians}$$

= 0.4089 radians

5 Convert $87^0 19' 57''$ into Radian.

$$19'57'' = 19' + \frac{57''}{60}$$

$= 19' + 0.95'$

$= 19.95'$

$$87°19.95' = 87° + \frac{19.95'}{60}$$

$= 87° + 0.332° = 87.33°$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$87.33° = \frac{\pi}{180} \text{ x } 87.33 \text{ radian}$$

$= 1.524$ radian

6 Convert $67°11'43''$ into Radian

$$11'43'' = 11' + \frac{43''}{60}$$

$= 11' + 0.716'$

$= 11.72'$

$$67°11.72' = 67° + \frac{11.72'}{60}$$

$= 67° + 0.195°$

$= 67.2°$

$$1° = \frac{\pi}{180} \text{ radian}$$

$$67.2° = \frac{\pi}{180} \text{ x } 67.2 \text{ radian}$$

$= 1.173$ radian

7 Convert $\frac{4}{7}\pi$ radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7}\pi \text{ radian} = \frac{180}{\pi} \text{ x } \frac{4}{7}\pi \text{ degree}$$

= 102.9 degree

$= 102^0\ 0.9 \text{ x } 60'$

$= 102^0 54'$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \text{ x } 0.8357 \text{ degree}$$

$= 47.88^0$

$= 47^0\ 0.88 \text{ x } 60'$

$= 47^0 52.80'$

$= 47^0 52' 0.8 \text{ x } 60''$

$= 47^0 52' 48''$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \text{ x } 2.752 \text{ degree}$$

$= 157.7^0$

$= 157.7^0 \text{ x } 60'$

$= 157^0 42'$

10 Convent $\frac{3}{5}\pi$ radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5}\pi \text{ radian} = \frac{180}{\pi} \text{ x } \frac{3}{5}\pi \text{ degree}$$

$= 108^0$

Assignment

Convert into Degree

1 12 Radian

Convert into Radians

2 78^o

3 $47^0 20'$

4 $52^0 36' 45''$

5 $25^0 38''$

Convert into degree, minute and seconds

6 46.723^o

7 68.625^o

8 0.1269 Radian

9 2.625 Radians

10 3/5 Radian

Trigonometry - Trigonometrical ratios

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios $\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}$ and $\frac{AC}{BC}$ do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB", BC" and AC".

For the angle

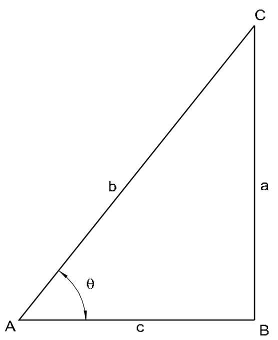
AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios

Fig 1



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The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\tan\theta}$$

$$\sin \theta = \frac{\text{sideBC}}{\text{sideAC}} = \frac{a}{b}$$

$$\cos \theta = \frac{\text{side AB}}{\text{sideAC}} = \frac{c}{b}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} x \frac{b}{c} = \frac{a}{c}$$

$$= \frac{\text{side BC}}{\text{side AB}} = \tan \theta$$

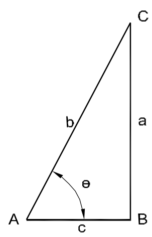
$$\sin \theta = \frac{1}{\text{cosec } \theta} \text{ or cosec } \theta = \frac{1}{\sin \theta} \text{ or } \sin\theta.\text{cosec } \theta = 1$$

$$\cos\theta = \frac{1}{\sec \theta} \text{ or } \sec \theta = \frac{1}{\cos \theta} \text{ or } \cos \theta . \sec\theta = 1$$

$$\tan \theta = \frac{1}{\cot \theta} \text{ or } \cot \theta = \frac{1}{\tan \theta} \text{ or } \cot \theta . \tan \theta = 1$$

By pythogoras theorem we have, $AC^2 = AB^2 + BC^2$

Fig 2



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Dividing both sides of the equation by AC^2, we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[\frac{AB}{AC}\right]^2 + \left[\frac{BC}{AC}\right]^2$$

$1 = (\cos \theta)^2 + (\sin \theta)^2$

$\sin^2 \theta + \cos^2 \theta = 1$

Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios

$\tan \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta}$ and $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

It can be transformed as

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\text{or } \cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

We know $\sin^2\theta + \cos^2\theta = 1$

Dividing both sides by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\text{or } 1 + \tan^2\theta = \sec^2\theta$$

Using the same equation

$$\sin^2\theta + \cos^2\theta = 1.$$

Dividing both sides by $\sin^2 q$,

$$\frac{\text{Sin}^2\theta}{\text{Sin}^2\theta} + \frac{\text{Cos}^2\theta}{\text{Sin}^2\theta} = \frac{1}{\text{Sin}^2\theta}$$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2 q = \text{cosec}^2 q$$

$$1 + \tan^2 q = \sec^2 q$$

Trigonometrical Tables

Ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

When q increases,

Sine value increases;

Cosine value decreases;

Tangent value increases to more than 1 when the angle is more than 45° (tan60° = 1.732)

Sine of an angle = Cosine of its complementary angle

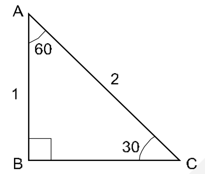
Cosine of an angle = Sine of its complementary angle

Examples

If $\sin 30^0 = \frac{1}{2}$ find the value of $\sin 60^0$

By applying pythagores theorem

Fig 3



$BC^2 = AC^2 - AB^2$

$BC^2 = 2^2 - 1^2$

$= 4 - 1$

$= 3$

$BC = \sqrt{3}$

$\sin 60^0 = \frac{\sqrt{3}}{2}$

$\text{Cos}\theta = \frac{3}{5}$ Find the other trigonometrical ratios

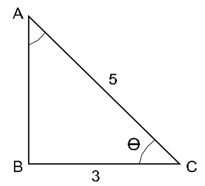
By applying pythagores theorem

$AB^2 = AC^2 - BC^2$

$= 5^2 - 3^2 \quad = 25 - 9$

$= 16$

Fig 4



$AB = \sqrt{16} = 4$

Now $\sin\theta = \frac{4}{5}$

$\tan\theta = \frac{4}{3}$

$\text{Cosec}\ \theta = \frac{5}{4}$

$\sec\ \theta = \frac{5}{3}$

$\cot\ \theta = \frac{3}{4}$

Signs of trigonometrical functions for angles more than 90^0

Ratio	90 - θ	90 + θ	180 - θ	180 + θ	270 - θ	270 + θ	360 - θ	- θ
sin	cos	cos	sin	- sin	- cos	- cos	- sin	- sin
cos	sin	- sin	- cos	- cos	- sin	sin	cos	cos
tan	cot	- cot	- tan	tan	cot	- cot	- tan	- tan
cosec	sec	sec	cosec	- cosec	- sec	- sec	- cosec	- cosec
sec	cosec	- cosec	- sec	- sec	- cosec	cosec	sec	sec
cot	tan	- tan	- cot	cot	tan	- tan	- cot	- cot

Simplify :

cot θ + tan (180+θ) + tan(90-θ) + (tan 360 - θ)

= cot θ + tan θ - cot θ - tan θ

= 0

Simplify :

$$\frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cos(90-\theta)}$$

$$= \frac{(-\sin\theta)\,x\,(\sec\theta)\,x\,(-\tan\theta)}{(\sec\theta)\,x\,(-\sin\theta)\,x\,(-\sin\theta)}$$

$$= \frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta} = \sec\theta$$

simplify:

$$\frac{\cos(90°+\theta)\sec(-\theta)\tan(180°-\theta)}{\sec(360°-\theta)\sin(180°+\theta)\cot(90°-\theta)}$$

cos (90^0 + θ) = - sin θ

sec (- θ) = sec θ

tan (180^0 - θ) = - tan θ

sec (360^0 - θ) = sec θ

sin (180^0 + θ) = - sin θ

cot (90^0 + θ) = - tan θ

$$\frac{\cos(90°+\theta)\sec(-\theta)\tan(180°-\theta)}{\sec(360°-\theta)\sin(180°+\theta)\cot(90°-\theta)}$$

$$= \frac{(-\sin\theta)(\sec\theta)(\tan\theta)}{(\sec\theta)(-\sin\theta)(-\tan\theta)}$$

= 1

Simplify:

Cot θ + tan (180^0 + θ) + tan (90^0 + θ) + tan (360^0 - θ)

tan (180^0 - θ) = tan θ

tan (90^0 + θ) = - cot θ

tan (360^0 - θ) = - tan θ

cot θ + tan (180^0 + θ) + tan (90^0 + θ) + tan (360^0 - θ)

cot θ + tan θ - cot θ - tan θ = 0

Assignment

1 Given sin 30^0 = 1/2, find the value of tan 60^0

2 If cos θ = 4/5, find the other radios

3 If sin A = 3/5, find cos θ, tan θ & sec θ

4 If tan θ = 24/7, find sin θ and cos θ

5 Find the value of cos θ and tan θ, if sin θ = 1/2

6 If cos θ = 5/13, find the value of tan θ

7 If sin θ = 1/2, find the value of $\sin^2\theta - \cos^2\theta$

8 What is the value of

$$\frac{\sin^2 30°}{\cos^2 45°} + \frac{\tan 45°}{\sec 60°} - \frac{\sin 60°}{\cot 45°} - \frac{\cos 30°}{\sin 90°}$$

Simplify :

1 tan (90 + A) + (tan 180 + A) tan (90 + A)

2 $$\frac{\cos(90+\theta)\cdot\sec(-\theta)\cdot\tan(180-\theta)}{\sec(360+\theta)\cdot\sin(180+\theta)\cdot\cot(90+\theta)}$$